MULTI-OBJECTIVE OPTIMIZATION OF REINFORCED CEMENT CONCRETE RETAINING WALL

A thesis submitted in partial fulfillment of the requirements for the degree of

Bachelor of Technology
In
Civil Engineering

By

Sandip Purohit
110CE0051

Under The Guidance of
Dr. Sarat Kumar Das

Department of Civil Engineering
National Institute of Technology Rourkela
Orissa -769008, India

May 2014

DEPARTMENT OF CIVIL ENGINEERING
NATIONAL INSTITUTE OF TECHNOLOGY
ROURKELA, ODISHA-769008

CERTIFICATE

This is to certify that the thesis entitled, **"MULTI-OBJECTIVE OPTIMIZATION OF REINFORCED CEMENT CONCRETE RETAINING WALL"** submitted by **Sandip Purohit** bearing roll no. **110CE0051** of C**ivil Engineering Department,** National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

Rourkela
Date: 10th May, 2014

Prof. Sarat Kumar Das
Department of Civil Engineering
National Institute of Technology
Rourkela, Odisha-769008

ACKNOWLEDGEMENTS

I would like to express my deep sense of gratitude to my supervisor **Dr. Sarat Kumar Das** for giving me this opportunity to work under him even at a time when he had too many commitments. I am very thankful to him for the patience he had with me. He's been the most pivotal person for the execution of this project. An exceptional individual—extremely industrious and bright, it's really inspiring to see his energy levels.I shall remain ever indebted to him for his unrelenting support and guidance.

I am thankful to **Prof. N. Roy**, Head of the department, Department of Civil Engineering for providing us with necessary facilities for the research work.

I am grateful to Geotechnical Engineering Laboratories for providing me with full laboratory facilities. I express my thanks to all the laboratory staff for their constant help and support at the time of need.

I also thank Rupashree and Surabhi ma'am and Partha Sir for their valuable advice and intake into my project.

Finally, my grateful regards to my parents who have always been so supportive for my academic pursuits.

Sandip Purohit
110CE0051
Department of Civil Engineering
National Institute of Technology, Rourkela
Rourkela

ABSTRACT

The optimum design of reinforced cement concrete cantilever (RCC) can be solved in the for the minimum cost satisfying required external and internal stability criteria. For high level decision making, an ideal optimization should give the optimized cost vis-a-vis corresponding factor of safety (FOS) against external stability like bearing, sliding and overturning, which is known as multi-objective optimization problem. In the present work multi-objective optimization of the RCC retaining wall is presented with conflicting objectives of minimum cost and maximum factor of safety against external stability. The Pareto-optimal front is presented using an evolutionary multi-objective optimization algorithm, non-dominated sorting genetic algorithm (NSGA-II). The results are compared with that obtained using single objective optimization of minimizing the cost. Based on the results a guideline for the optimum dimensioning of the RCC cantilever retaining wall is presented.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

Chapter 1| **INTRODUCTION**

In structed for supporting a vertical or nearly vertical earth back fill. The other uses of retaining development of roads with constrained inland space in permanent ways, retaining walls is generally conwall include hill side roads, elevated and depressed roads, canals, erosion protection, bridge abutments, etc. The reinforced cement concrete cantilever (RCC) retaining wall is the most common type of retaining wall used in such cases. The design of RCC retaining wall is a trial and error process, in which a trial design with its geometry is proposed (may be as per existing guideline) and checked against different stability criteria [31]. Very often it is an over designed wall with hardly any consideration for optimum dimension. However, the economy is an essential part of a good engineering design and needs to be considered explicitly in design to obtain an optimum section.

The optimum section of a retaining wall can be considered in the framework of an optimization problem and can be solved using the optimization techniques. Keskar and Adidam [19] have used an interior penalty function based nonlinear optimization technique (Deb [12]) for the design of a cantilever retaining wall. Saribas and Erbatur [33] used separate optimization models to find out optimum cost and minimum weight of the cantilever retaining wall using interior penalty functions. Castillo et al. [7], Low [24] and Babu and Basha [3] discussed the optimum design of retaining wall using reliability based method.

Methods for developing low-cost and low-weight designs of reinforced concrete retaining structures have been the subject of research for many years (Fang et al. [16]; Rhomberg and Street [31]; Alshawi et al. [2]; Keskar and Adidam [19]; Saribaş and Erbatur [33]; Low et al. [25]; Chau and Albermani [9]; Bhatti [4]; Babu and Basha [3]). However, the application of heuristic and evolutionary methods to the design of retaining structures is relatively new: Ceranic et al. [8] and Yepes et al. [36] applied simulated annealing (SA); Ahmadi-Nedushan and Varaee [1] used particle swarm optimization (PSO); and Kaveh and Abadi [18] applied harmony search. Recently, Camp and Akin [5] discussed the optimum design of retaining wall using an evolutionary algorithm, big-bang big crunch (BBBC) algorithm. Although the research into the design of retaining structures using evolutionary methods is limited, there are numerous studies on their application to reinforced concrete structures. Coello Coello et al. [11], Rafiqa and Southcombea [29], Rajeev and Krishnamoorthy [30], Camp et al. [6], Lee and Ahn [22], Lepš and Šejnoha [23], Sahab et al. [32], Govindaraj and Ramasamy [17], and Kwak and Kim [20,21] all applied various forms of GAs to the cost-optimization problem. Paya et al. [26], Perea et al. [28], and Paya-Zaforteza et al. [27] optimized reinforced concrete structures using simple and hybrid SA algorithms. Effects of height of the retaining wall, backfill soil parameters on the optimum size of the wall also have been discussed. In all the above work, the optimization problem has been framed with a single objective of minimizing cost, satisfying the stability against external stability criteria. For high level decision making, an ideal optimization should give the optimized cost vis-a-vis corresponding factor of safety (FOS) against external stability like bearing, sliding and overturning. Hence the more generic problem with a retaining wall is to minimize the cost and to maximize factor of safety (FOS) against external stability. Such type practical optimization problems with more than one conflicting objectives like minimizing the cost and maximizing the FOS against bearing, sliding and overturning is known as multi-objective optimization or vector optimization.

In contrast to single-objective optimization, a solution to a multi-objective problem does not contain single global solution, and may contain many numbers of feasible solutions (including all optimal solutions) that fit a predetermined definition for an optimum. The predominant concept in defining an optimal point is that of Pareto optimality (Deb [12]). In the traditional optimization methods, multi-objective problems are considered one at a time and other objectives are considered as constraints. Hence, in such cases Pareto optimal (trade-off) solutions are obtained with number of runs of the problem. Fig. 1 shows the difference in traditional and evolutionary multi-objective optimization algorithm (EMO). It can be seen that in case of traditional multi-objective optimization, it is converted to single objective optimization problem with importance attached to each objective or taking other objectives as constraints. But an ideal multi-objective algorithm should find out a set of Pareto optimal solution considering all the objectives as equally important. Then one of the solutions is chosen considering higher level information at the decision making level. Population-based evolutionary multi-objective optimization (EMO) is able to generate the required Pareto front in a single run. A comprehensive review of EMO algorithms can be found in Deb [12] and Coello et al. [10]. But, application of multi-objective optimization is limited in geotechnical engineering (Deb and Dhar [13]; Deb et al. [14]). Deb and Dhar [13] proposed a combined simution-optimization-based methodology to identify the optimal design parameters for granular bed–stone column-improved soft soil. Deb et al. [14] extended the same multi-objective optimization methodology to access the stability of embankments constructed over clays improved with the combined use of stone columns and geosynthetic reinforcements.

Hence, in this paper the optimum design of retaining wall is presented in a single and multi-objective framework. For comparison, the optimization of retaining wall using a simple optimization tool based on traditional method is presented in the first step. Then the optimization results of retaining wall in multi-objective framework using Elitist nondominated sorting genetic algorithm (NSGA-II) (Deb [12]) are discussed.

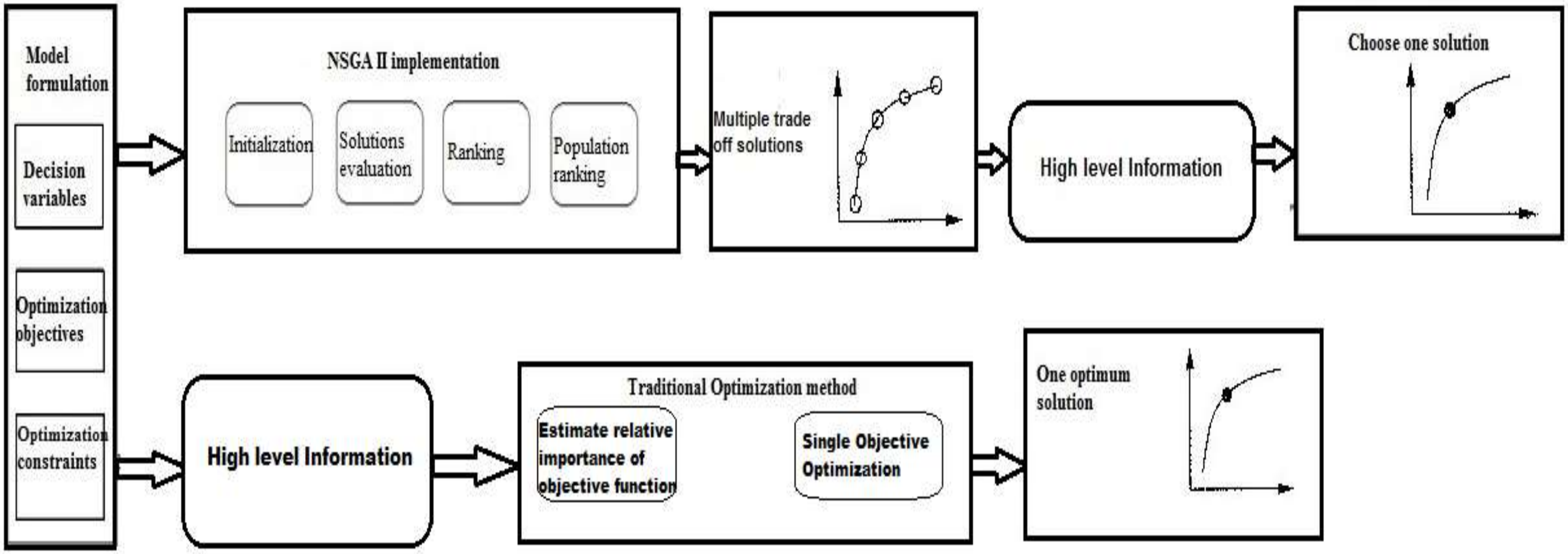

Fig. 1. Typical diagram showing variation between traditional and Evolutionary Multi-objective optimization model

Chapter 2| **ANALYSIS AND METHODOLOGY**

The analysis consists of (i) formulation of the optimization problem and (ii) solution of the optimization problem using traditional and genetic algorithms. The formulation of the optimization problem and its solutions are presented as follows.

2.1 Formulation of the objective function

In the present formulation for the single objective optimization problem, the total cost (to be minimized), which consists of cost of concrete and cost of steel reinforcement, is considered.

Objectives

Minimize Total Cost (TC) per meter run

$TC = f\,(L_h, L_t, S, b, t, p_{tt}, p_{th}, p_{ts})$

$$TC = c_c Q_c + c_r W_{st} \qquad (1)$$

Where c_c, c_r are the unit rate of concrete and steel reinforcement respectively and the rates are taken from the Delhi Schedule of Rates 2007 (DSR -2007 [15]). Q_c and W_{st} are the volume of concrete and weight of reinforcement steel, respectively. The cost of shuttering is not considered keeping in mind its effect is minimal in the total cost and it depends on the volume of the concrete. However, if desired it can be considered for the optimum cost design.

The geometric parameters of the retaining wall like top width of stem, heel projection, toe projection and their thickness, percentage of the reinforcement in base slab and stem are considered as the design variables which are varied to reach the optimum cost.

The above variables are presented in Fig. 2 and are described as follows:

L_h = projection of heel from the base of the stem;

L_t = projection of toe from the base of the stem;

b = width of the batter of back face of the wall;

S = width of stem at top;

t = thickness of the base slab;

$p_{th} =$ reinforcement percentage in heel slab;

$p_{ts} =$ reinforcement percentage in stem of the wall;

p_{tt} = reinforcement percentage in toe slab;

Constraints:

The constraints are considered in terms of criteria for external and internal stability of retaining wall.

The different constraints considered in terms of factor of safety (FOS) are as follows:

External stability

FOS against overturning $FS_{ot} \geq 2.0$

FOS against sliding $FS_{sli} \geq 1.5$

FOS against eccentricity $FS_{e} \geq 1.0$

FOS against bearing $FS_{b} \geq 3.0$

Internal stability

As RCC cantilever retaining wall is being considered in this paper, the internal stability in terms of flexure and shear failure are calculated based on IS: 456 -2000 (Indian standard specification for plain and reinforced concrete) using limit state method.

FOS against toe shear failure $FS_{tsh} \geq 1.5$

FOS against toe moment failure $FS_{tm} \geq 1.5$

FOS against heel shear failure $FS_{hsh} \geq 1.5$

FOS against heel moment failure $FS_{hm} \geq 1.5$

FOS against stem shear failure $\mathrm{FS_{hsh}} \geq 1.5$

FOS against stem moment failure $FS_{sm} \geq 1.5$

The details of external and internal stability analysis considered for the proposed study is presented here as followings.

2.2 The detailed analysis of constraints

The detailed formulation of the constraints is described as follows.

2.2.1External stability

Overturning Failure Mode

The general layout of the retaining wall with its geometry and different forces acting on it are shown in Fig. 2.

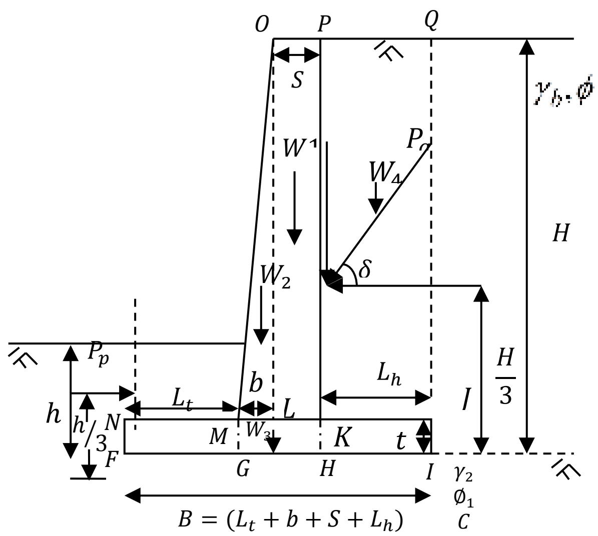

Fig. 2. General layout of a RCC cantilever retaining wall showing dimensions and different forces acting on it.

The total moment of resistance ΣM_R can be expressed as

$$\sum M_R = M_1 + M_2 + M_3 + M_4 \tag{2}$$

Where

$$M_1 = \gamma_c (H - t) S [L_t + b + S / 2] \tag{3}$$

$$M_2 = \frac{1}{2} \gamma_c (H - t) b \left[L_t + \frac{2}{3} b \right] \tag{4}$$

$$M_3 = \frac{1}{2} \gamma_c t (L_t + b + S + L_h)^2 \tag{5}$$

$$M_4 = \gamma_b (H - t) L_h [S + b + L_t + L_h / 2] \quad (6)$$

The overturning moment can be described as

$$\sum M_o = M_5 \quad (7)$$

$M_5 = P_a (H / 3)$, where $P_a =$ Rankine's total active earth pressure (8)

$$FS_{ot} = \frac{\sum M_R}{\sum M_o} \quad (9)$$

Where

$FS_{ot} =$ factor of safety against overturning;

$\gamma_c =$ unit weight of concrete;

$\gamma_b =$ unit weight of backfill soil;

$L_h =$ projection of heel from the base of the stem;

$L_t =$ projection of toe from the base of the stem;

b = width of the batter of back face of the wall;

S = width of stem at top;

H = height of the retaining wall;

t = thickness of the base slab;

Sliding Failure Mode

$$\sum F_R = \sum V \text{ x} \tan((\frac{2}{3})\phi) + (\mathrm{S+ b+ L}_t + \mathrm{L}_h)(\frac{2}{3})\mathrm{c} + \mathrm{P}_p \quad (10)$$

$$\sum F_D = P_a \quad (11)$$

$$FS_{sli} = \frac{\sum F_R}{\sum F_D} \quad (12)$$

Where

$FS_{sli} =$ factors of safety against sliding;

$\sum V =$ sum of the vertical forces acting on retaining wall;

$\phi_2 =$ friction angle of the soil below the foundation;

c = unit cohesion of soil below the base slab of the retaining wall;

$P_p =$ Rankine's total passive earth pressure from toe side of the wall;

Eccentricity failure mode

$$FS_e = \frac{B/6}{e} \tag{13}$$

$FS_e =$ factors of safety against eccentricity;

e = eccentricity;

B = width of base slab;

Bearing Failure Mode

$$q_u = cN_c d_c i_c + qN_q d_q i_q + 0.5\gamma_2 (B - 2\mathrm{e})\,\mathrm{N}_\gamma\, d_\gamma i_\gamma \tag{14}$$

$$q_{\max} = \left[\frac{\sum V}{B}\left(1 + \frac{6e}{B}\right) \right] \tag{15}$$

$$FS_b = \frac{q_u}{q_{\max}} \tag{16}$$

Where

$FS_b =$ factor of safety against bearing;

$d_c, d_q, d_\gamma =$ depth factors;

$i_c, i_q, i_\gamma =$ load inclination factors;

$N_c, N_q, N_\gamma =$ bearing capacity factors;

The variation of bearing pressure as obtained above for the heel and toe slab is shown in Fig. 3.

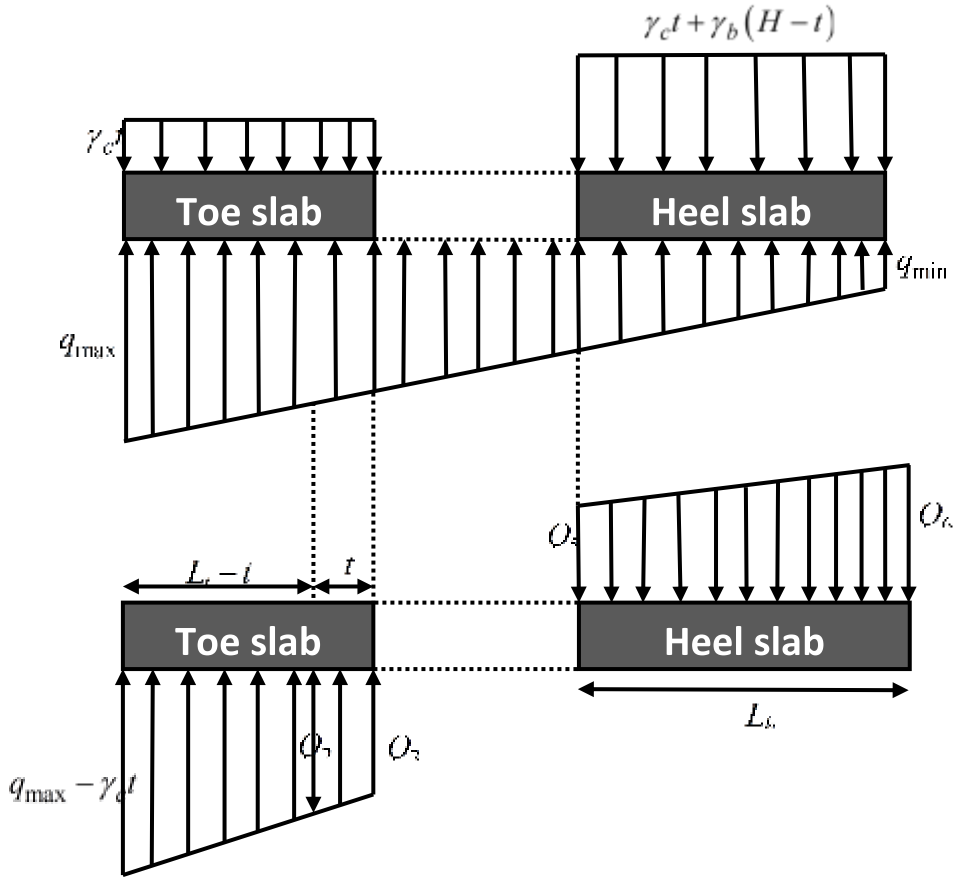

Fig. 3. Variation of bearing pressure

2.2.2 Internal stability

The FOS in terms of structural design of heel slab, toe slab and stem slab for shear and bending is presented as follows:

Toe Shear Failure Mode

$$V_{utoe} = \frac{1}{2}\left[\left(q_{\max} - \gamma_c t\right) + Q_2\right]\left(L_t - t\right) \tag{17}$$

$$\tau_{vtoe} = \frac{V_{utoe}}{pt} \tag{18}$$

$$FS_{tsh} = \frac{\tau_c}{\tau_{vtoe}} \tag{19}$$

$\tau_{vheel} =$ nominal shear stress in heel slab;

$\tau_{vstem} =$ nominal shear stress in stem of the wall;

$\tau_{vtoe} =$ nominal shear stress in toe slab;

Toe moment Failure Mode

$$M_{utoe} = \frac{1}{2}\left(q_{\max} - \gamma_c t + Q_3\right) L_t \left(\frac{Q_3 + 2\left(q_{\max} - \gamma_c t\right)}{Q_3 + \left(q_{\max} - \gamma_c t\right)}\right)\left(\frac{L_t}{3}\right) \tag{20}$$

$$MR_{toe} = 0.87 f_y A_{stoe} t\left[1 - \frac{A_{stoe}}{pt}\frac{f_y}{f_{ck}}\right] \tag{21}$$

$$FS_{tm} = \frac{MR_{toe}}{M_{utoe}} \tag{22}$$

Where

$FS_{tm} =$ factors of safety against toe shear failure;

Q_2 = net intensity of pressure at a section at a distance t from stem base;

Q_3 = net intensity of pressure at a section at the base of stem;

Heel Shear Failure Mode

$$V_{uheel} = \frac{1}{2}\left[Q_6 + Q_5\right] L_h \tag{23}$$

$$\tau_{vheel} = \frac{V_{uheel}}{pt} \tag{24}$$

$$FS_{hsh} = \frac{\tau_c}{\tau_{vheel}} \tag{25}$$

Heel Moment Failure Mode

$$M_{uheel} = \frac{1}{2}\left(Q_5 + Q_6\right) L_h \left[\left(\frac{2Q_6 + Q_5}{Q_5 + Q_6}\right)\frac{L_h}{3}\right] = \frac{L_h^2}{6}\left[2Q_6 + Q_5\right] \tag{26}$$

$$MR_{heel} = 0.87 f_y \left(\frac{A_{sheel}}{pt}\left(pt\right)\right) t\left[1 - \frac{A_{sheel}}{pt}\frac{f_y}{f_{ck}}\right] \tag{27}$$

$$FS_{hm} = \frac{MK_{heel}}{M_{uheel}} \tag{28}$$

Stem Shear Failure Mode

$$V_{ustem} = K_a \gamma_b \frac{\left(H - S\right)^2}{2} \tag{29}$$

$$\tau_{vstem} = \frac{V_{ustem}}{pS} \tag{30}$$

$$FS_{ssh} = \frac{\tau_c}{\tau_{vstem}} \tag{31}$$

Stem Moment Failure Mode

$$M_{ustem} = K_a \gamma_b \frac{H^2}{2}\left(\frac{H}{3}\right) = \frac{1}{6} K_a \gamma_b H^3 \tag{32}$$

$$MR_{stem} = 0.87 f_y \left(\frac{A_{sstem}}{pS}(pS)\right) S \left[1 - \frac{A_{sstem}}{pS}\frac{f_y}{f_{ck}}\right] \tag{33}$$

$$FS_{sm} = \frac{MR_{stem}}{M_{ustem}} \tag{34}$$

2.3 Evolutionary Multi-objective (EMO) Algorithm, (NSGA-II)

The non-dominated sorting genetic algorithm (NSGA), which was proposed by Srinivas and Deb[34], had shortcomings like high computational complexity of non dominated sorting, lack of elitism and need for specifying the sharing parameter. All those issues were overcome in NSGA-II, which is a simple constraint handling EA. It is efficient in handling both single and multi-objective problems.

NSGA-II has some improved features such as fast non-dominated sorting procedure, an elitist-preserving approach and a parameter less niching operator. The brief description of NSGA-II is discussed below and details can be found in Deb [12]. The major steps in implementation of NSGA-II can be described as population initialization, non-dominated sort, crowding distance, selection and GA parameters- crossover and mutation. A flowchart showing the NSGA-II algorithm is presented in fig. 1.

The population is initialized based on the problem range and constraints. Then solution for each objective is found. It is sorted into each front on the basis of non-domination using fast sort algorithm. The first front is completely non-dominant set in the current population and second front is dominated by the individuals in the first front only and so on. Fitness value of each individual is evaluated and they are assigned rank **accordingly** to absolute normalized difference in the function values of two adjacent solutions.

Once the non-dominated sort is complete, the crowding-distance is calculated for each individual. The crowding-distance measures how close an individual is to its neighbors. The crowding computation requires sorting the population according to each objective function value in ascending order of magnitude. Then, for each objective function, the boundary solutions (solution with smallest and largest function values) are assigned an infinite distance value. For other intermediate solutions, they are assigned a distance value equal to the absolute normalized difference in the function values of two adjacent solutions. This is continued with other objective functions also. The overall crowding-distance value is then calculated as the sum of individual distance values corresponding to each objective. Crowding-distance sort is done in order to maintain the diversity in the population. Diversity is an important aspect in EA.

Parents are selected from the population by using binary tournament selection with crowded-comparison operator which is based on the rank and crowding distance. An individual is selected if the rank is lesser than the other or if crowding distance is greater than the other (crowding distance is compared only if the ranks for both individuals are same). The selected population generates offspring from crossover and mutation operators. The NSGA II uses Simulated Binary crossover (SBX) operator for crossover and polynomial mutation. During the process, elitism is ensured by combining the offspring population with the parent population and then selecting individuals for the next generation. Again the combined population was sorted according to non-domination. Then new generation was filled by each front subsequently until the population size exceeds the current population size. The above process is repeated to generate the subsequent generations.

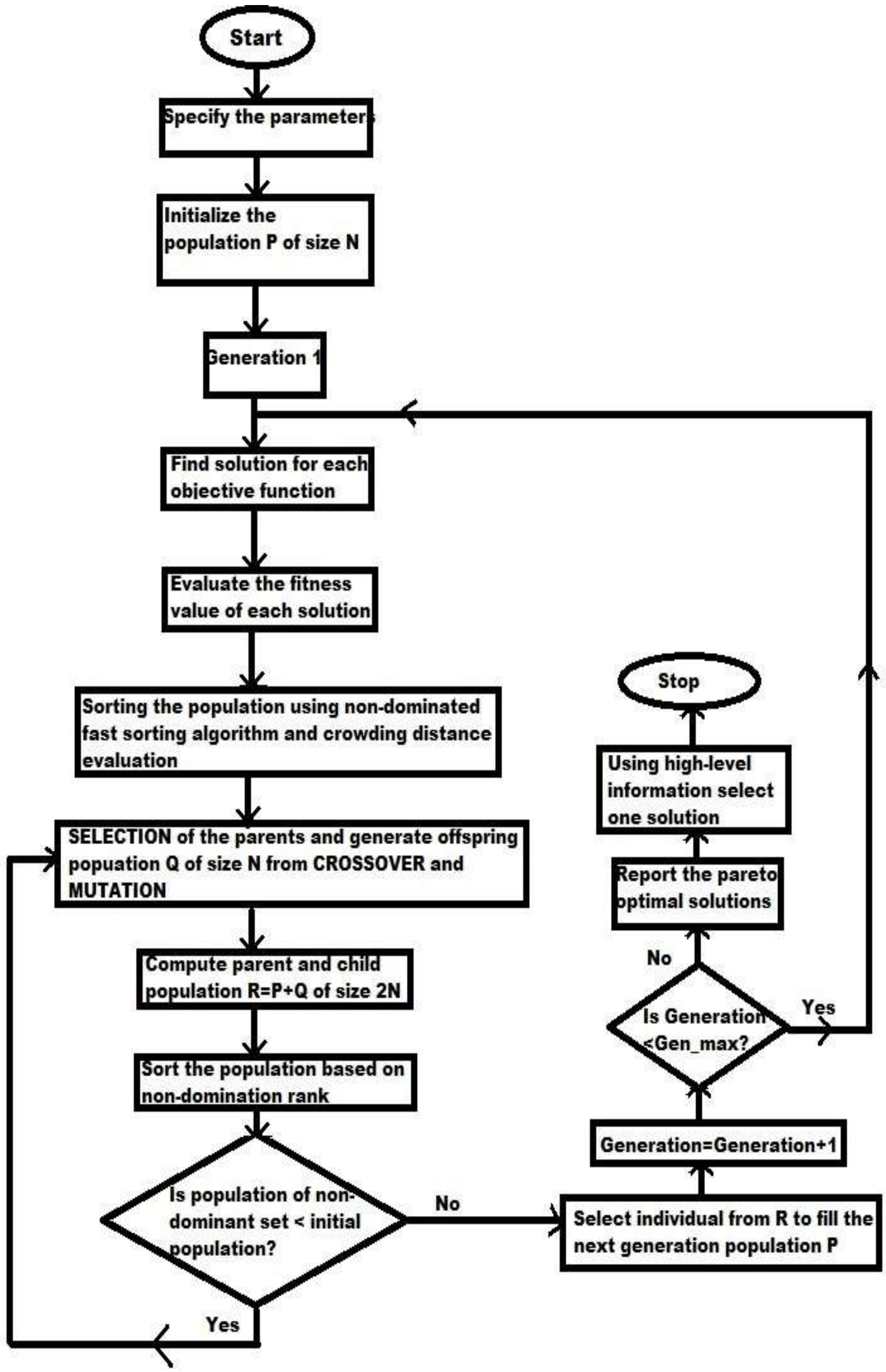

Fig. 4. Flowchart showing the working principle of NSGA II

As discussed earlier, the optimized design of retaining wall is considered in both single and multi-objective optimization framework. Hence, the single and multi-objective optimization results are presented and discussed separately. In case of single objective optimization the minimization of cost is taken as the objective with both external and internal stability criteria as constraints as discussed in the previous section.

3.1 Single objective optimization using genetic algorithm

In this section optimization of retaining wall using GA is presented and discussed. The retaining wall of height of 6.0m and $\phi = 30^0$, foundation soil parameter c = 40 kN/m^2 and angle of internal friction (ϕ) = 20^0 is considered. Fig. 5 shows the variation in total cost of the retaining wall of different heights and angle of internal friction of backfill soil.

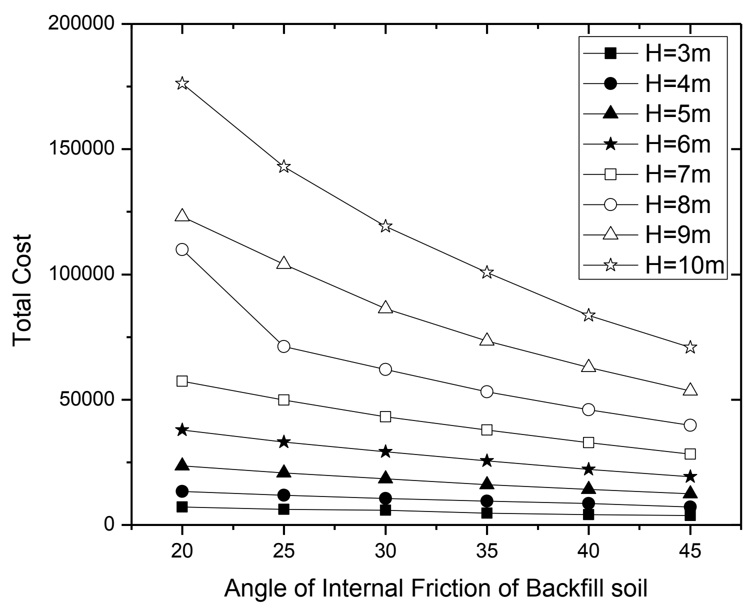

Fig. 5. Variation of Total Cost with Angle of Internal Friction of Backfill Soil

It can be seen that as expected the total cost increases with increase in height of the wall and decreased with increase in ϕ value. It can be seen that there is a significant decrease in the cost with ϕ value and height of wall beyond 5m. Hence, optimum dimension of retaining walls up to 5.0m can be considered as one group and walls above 5.0m can be considered as another group**.**

The variation in the FOS against the external stability criteria; overturning, sliding, eccentricity and bearing with different angle of internal friction of backfill soil for various heights are presented as follows. Fig. 6 shows the variation in FOS against overturning with ϕ value for different heights of the retaining wall.

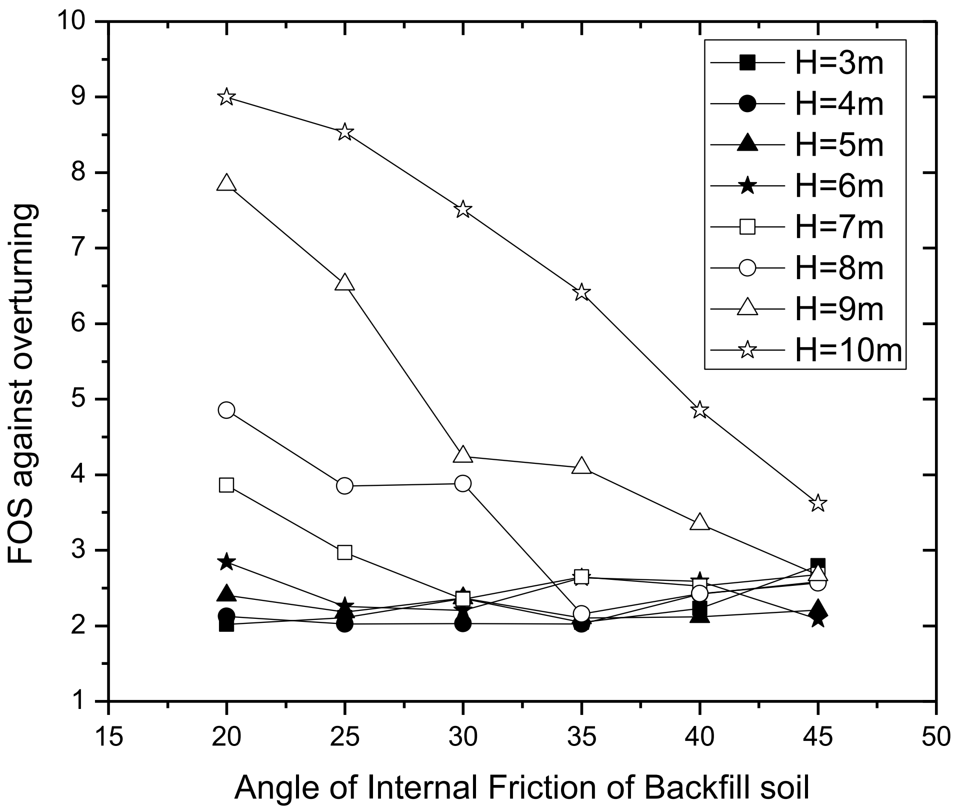

Fig. 6. Variation of FOS against overturning

From Fig. 6 it can be seen that for retaining walls of height 8m, 9m and 10m, there is a significant decrease in FOS against overturning with an increase in angle of internal friction. Retaining wall of height 7m shows moderate decrease in FOS whereas retaining wall of height 3m to 6m are arguably undeterred by an increase in the angle of internal friction. As discussed earlier, similar trends have also been observed while using MS Excel solver, but the numerical values are different. The variation of FOS against sliding is shown in Fig. 7.

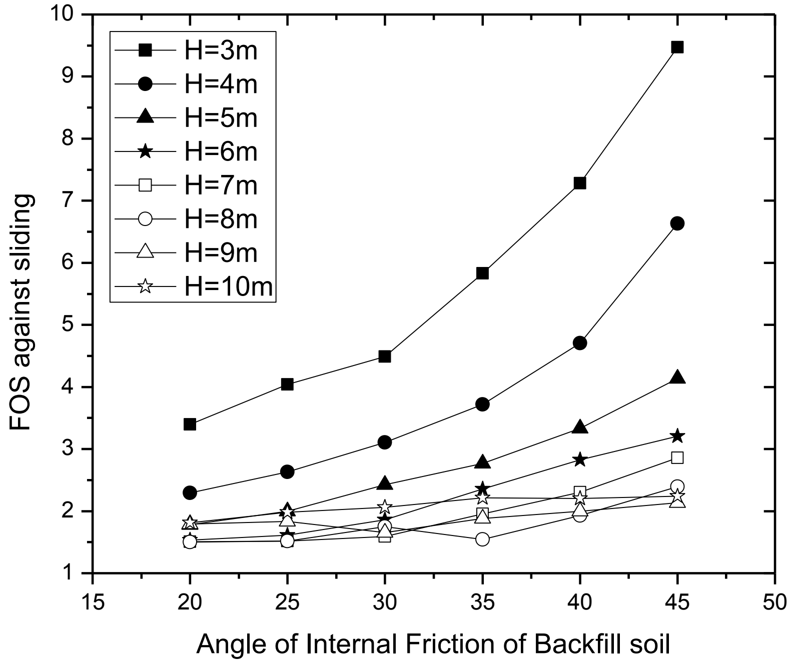

Fig. 7. Variation of FOS against Sliding

It can be seen that there is an considerable increase in FOS against sliding for retaining walls of height upto 4m and for ϕ greater than 30^0. However, for heights above 7m, the variation of the FOS against sliding is marginal due to the fact that the dimension of retaining wall becomes adequate and effect of ϕ value for the FOS against the sliding force is marginal.

The variation in FOS against eccentricity with ϕ value is presented in Fig. 8.

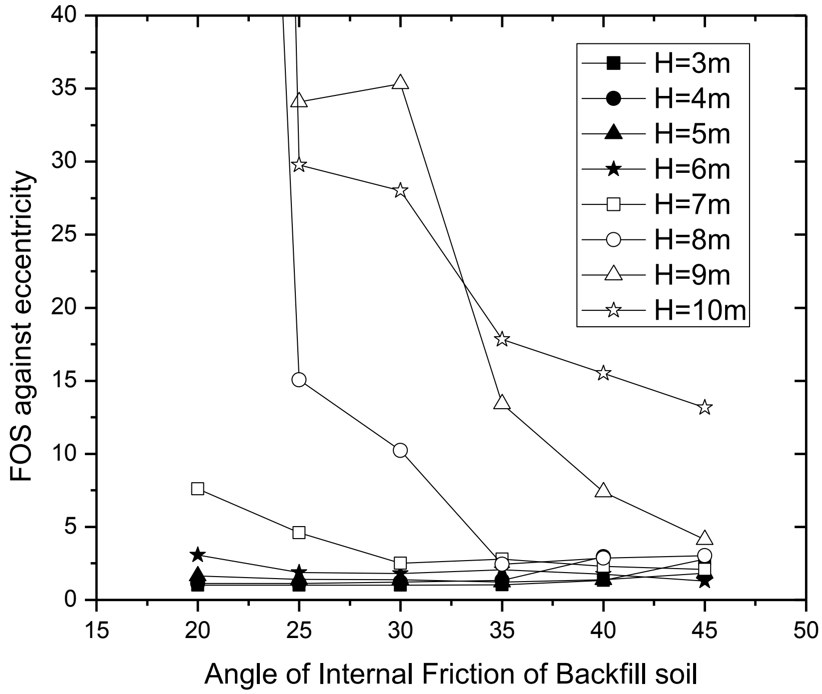

Fig. 8. Variation of FOS against Eccentricity

It can be seen that upto 6.0m height of retaining wall the FOS against eccentricity is important and decreased with increase in the angle of internal friction of backfill soil for retaining wall with heights upto 7m. This is due to the fact that thenafter other external stability criteria becomes important**.**

Fig. 9 shows the variation in FOS against bearing with angle of internal friction for retaining walls of different height.

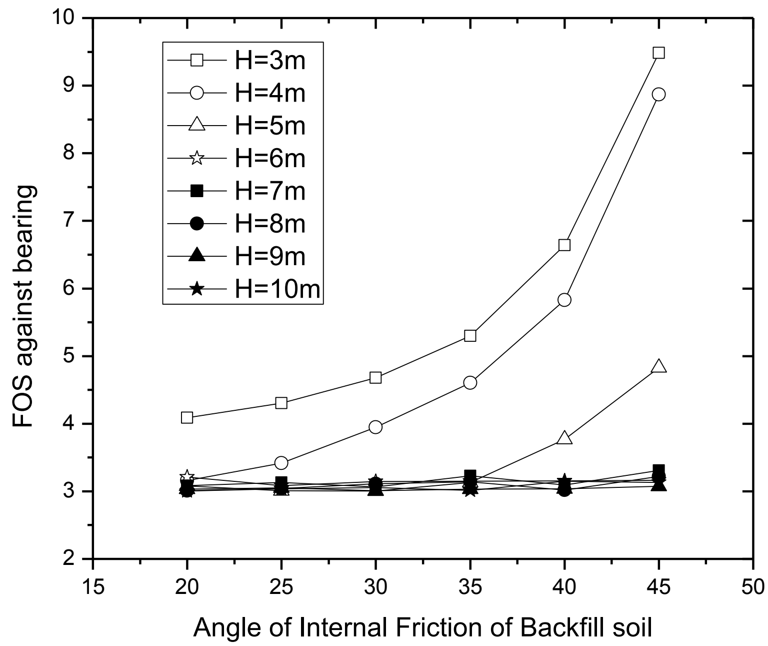

Fig. 9. Variation of FOS against Bearing

It is evident from Fig. 9 that there is a steep increase in the FOS against bearing with ϕ value for retaining walls upto height of 5m. The FOS against bearing decreased with increase in wall height. The increase is due to increase in overburden pressure. There is no appreciable change in FOS against bearing with increase in ϕ value for wall height more than 7m.

Table 1 Comparison of MS Excel Solver and genetic algorithm for wall height of 6m.

Variables	As per MS Excel Solver	NSGA-II	Remarks
Total cost (Rs)	32603.36	29221.78	10.3% savings in cost
L_t (m)	1.443	1.840	
L_h (m)	1.000	0.794	
s (m)	0.382	0.320	
b (m)	0.100	0.050	
t (m)	0.290	0.370	
p_{ts}(%)	0.695	0.700	
P_{th} (%)	0.200	0.150	
p_{tt} (%)	0.78	0.600	

For professional engineers the initial proportioning is very important for the design of cantilever retaining wall. Such a study is also made here to represent the geometry of retaining wall in terms of its height and results are presented in Table 3.

Table 2 Comparison of constraint violation as per MS Excel Solver and NSGA-II

Constraints	Constraint values	
	As per MS Excel Solver	NSGA-II
Overturning	0.416	0.204
Sliding	0.427	0.358
Bearing	0.0001	0.145
Eccentricity	1.013	0.996

Table 3 Optimum normalized dimensions of retaining wall of different height for different ϕ values

Φ (Degree)	Normalized Dimensions	Height of retaining wall (m)							
		3	**4**	**5**	**6**	**7**	**8**	**9**	**10**
20	**S/H**	0.05	0.06	0.07	0.08	0.08	0.08	0.09	0.1
	b/H	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.08
	L_t/H	0.33	0.25	0.3	0.33	0.31	0.23	0.22	0.2
	L_h/H	0.21	0.27	0.26	0.28	0.4	0.69	0.78	0.8
	t/H	0.06	0.04	0.05	0.05	0.06	0.08	0.06	0.07
25	**S/H**	0.05	0.06	0.07	0.07	0.07	0.08	0.08	0.08
	b/H	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.1
	L_t/H	0.33	0.25	0.28	0.34	0.33	0.3	0.21	0.2
	L_h/H	0.17	0.2	0.2	0.16	0.24	0.33	0.63	0.75
	t/H	0.04	0.04	0.05	0.06	0.06	0.06	0.05	0.07
30	**S/H**	0.03	0.05	0.06	0.06	0.07	0.07	0.07	0.08
	b/H	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	L_t/H	0.33	0.25	0.2	0.31	0.33	0.26	0.26	0.19
	L_h/H	0.17	0.17	0.22	0.13	0.13	0.31	0.33	0.59
	t/H	0.03	0.04	0.04	0.05	0.06	0.06	0.05	0.06
35	**S/H**	0.04	0.04	0.05	0.06	0.06	0.06	0.07	0.07
	b/H	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	L_t/H	0.17	0.25	0.2	0.21	0.26	0.3	0.23	0.17
	L_h/H	0.19	0.13	0.16	0.19	0.14	0.13	0.28	0.52
	t/H	0.03	0.04	0.04	0.05	0.05	0.06	0.06	0.05
40	**S/H**	0.03	0.04	0.04	0.05	0.05	0.06	0.06	0.06
	b/H	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	L_t/H	0.17	0.25	0.2	0.17	0.21	0.25	0.22	0.19
	L_h/H	0.17	0.13	0.13	0.18	0.14	0.1	0.18	0.3
	t/H	0.02	0.03	0.04	0.04	0.05	0.05	0.06	0.05
45	**S/H**	0.03	0.03	0.04	0.04	0.05	0.05	0.05	0.06
	b/H	0.02	0.04	0.01	0.01	0.01	0.01	0.01	0.01
	L_t/H	0.17	0.13	0.2	0.17	0.16	0.2	0.21	0.19
	L_h/H	0.17	0.14	0.1	0.11	0.14	0.11	0.11	0.17
	t/H	0.02	0.01	0.03	0.04	0.04	0.04	0.05	0.06

It was observed that S/H ratio increased with increase in height of retaining wall, but is almost independent of ϕ value. Though L_t/H does not vary much with height of the wall, but there is a substantial increase in L_h/H value with the height of the retaining wall for different ϕ value. The stem thickness ratio t/H varies from 0.04 to 0.07.

3.2 Multi-objective optimization using NSGA-II

The four objectives considered in the present study are; (i) minimize the total cost of construction (TC), maximize FOS against (ii) bearing, (iii) sliding and (iv) overturning, while satisfying the geotechnical and structural requirements as per Indian standard (IS 456: 2000).

$$\left.\begin{array}{l}
\text{(I) Minimize TC} \\
\qquad\qquad \mathrm{TC}= f\,(L_h,L_t,t,\, S,\, b,\, P_{ts},\, P_{th},P_{tt}) \\
\text{(II) Maximize FOS against bearing} \\
\qquad\qquad \mathrm{FS_b}=f(c,\, \phi,\, \gamma,\, L_h,L_t,t,\, S,\, b) \\
\text{(II) Maximize FOS against sliding} \\
\qquad\qquad \mathrm{FS_b}=f(c,\, \phi,\, \gamma,\, L_h,L_t,t,\, S,\, b) \\
\text{(II) Maximize FOS against overturning} \\
\qquad\qquad \mathrm{FS_b}=f(c,\, \phi,\, \gamma,\, L_h,L_t,t,\, S,\, b)
\end{array}\right\} \tag{35}$$

Constraints:

The constraints are the internal stability conditions and FOS against eccentricity.

The result of NSGA-II considering four objectives as described in Eqs. (35) with the design constraints as per the above design example is shown in Fig. 10 in terms of non-dominated front (Pareto solution).

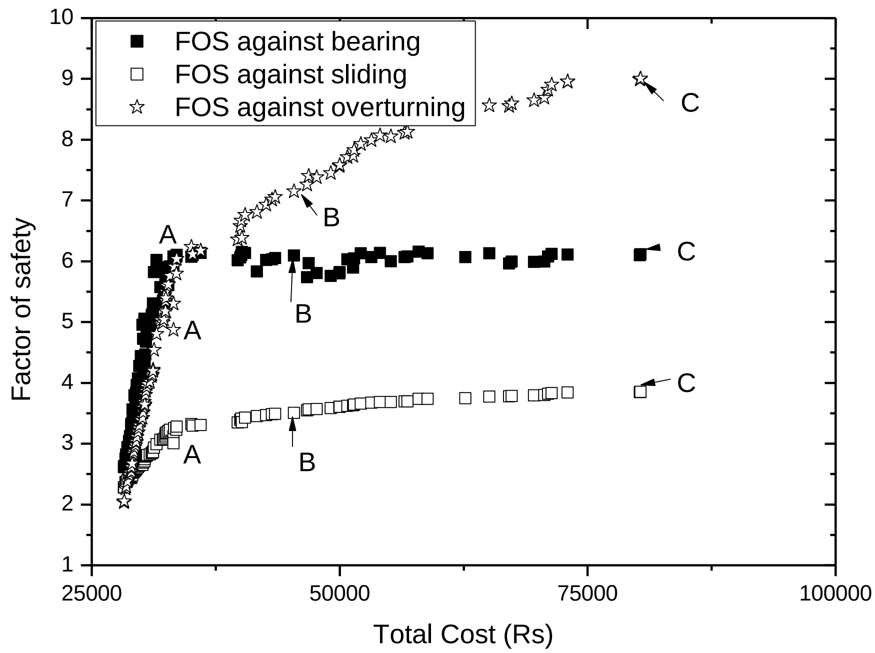

Fig. 10. Showing the variation of FOS (factor of safety) against bearing, sliding and overturning with total cost (For height of 6.0m and angle of internal friction of 30^0)

To validate the results three points A (32430.55, 5.54, 3.10, 5.19), B (45413.28, 6.09, 3.50, 7.15) and C (80327.79, 6.114, 3.85, 9.00), are chosen on the final front. In the parenthesis, the values represent the construction cost (Rs), FOS against bearing, sliding and overturning, respectively. In solution A, for FOS values of 5.54, 3.10, 5.19, corresponding construction cost is Rs 32430.55. Similarly for solution B, the cost increased to Rs.45413.28, and the corresponding FOS values are 6.09, 3.50 and 7.15, respectively for FOS against bearing, sliding and overturning. It can be seen that when FOS values are increased the cost increased, hence it is evident that when one objective is improved, the others need to be compromised as is expected for a multi-objective problem with conflicting objectives. Similar results can be derived from solution C. These trade-off solutions help in identifying the practical optimum design with higher level information in terms of governing criteria for selecting settlement, FOS or cost.

Table 4 The dimensions of the retaining wall and the percentage of reinforcement for the RW of 3m and ϕ of 20^0

L_t	L_h	t	S	B	P_{ts}	P_{th}	P_{tt}
1.947405	0.572081	0.838566	0.100127	0.057164	0.001200	0.001201	0.001200
1.961448	0.613721	0.958973	0.100154	0.056229	0.001200	0.001208	0.001201
1.000090	0.507783	0.802196	0.100000	0.054140	0.001200	0.001201	0.001200
1.427852	0.566843	0.802009	0.100006	0.056670	0.001201	0.001209	0.001201
1.847854	0.560442	0.803650	0.100030	0.058433	0.001200	0.001205	0.001200
1.454477	0.557109	0.802086	0.100017	0.055803	0.001200	0.001213	0.001200
2.000834	0.551556	0.893720	0.100812	0.070281	0.001201	0.001201	0.001328
1.315548	0.548511	0.801887	0.100001	0.050004	0.001200	0.001200	0.001200
1.994273	0.551556	0.880969	0.100356	0.070892	0.001201	0.001201	0.001203
1.677391	0.565627	0.802057	0.100001	0.054281	0.001200	0.001205	0.001201
1.320525	0.517735	0.802120	0.100001	0.056259	0.001200	0.001204	0.001201
1.138254	0.593313	0.802120	0.100004	0.056619	0.001200	0.001205	0.001201
1.950299	0.555455	0.862519	0.100131	0.066241	0.001200	0.001216	0.001200
1.936720	0.515975	0.802030	0.100008	0.060008	0.001200	0.001223	0.001201
1.890099	0.659315	0.961646	0.100016	0.058933	0.001200	0.001241	0.001223
1.400397	0.500980	0.802129	0.100008	0.059661	0.001200	0.001205	0.001200
1.558314	0.528883	0.802056	0.100020	0.055861	0.001200	0.001216	0.001200
1.495769	0.518852	0.802258	0.100000	0.052822	0.001200	0.001209	0.001200
1.390396	0.506146	0.801999	0.100001	0.052761	0.001200	0.001204	0.001200
1.449819	0.543657	0.802086	0.100002	0.054535	0.001200	0.001200	0.001206
1.803505	0.577990	0.803586	0.100006	0.051906	0.001200	0.001204	0.001200
1.994273	0.551556	0.887784	0.100602	0.070892	0.001201	0.001200	0.001203
1.628020	0.535910	0.802057	0.100005	0.055133	0.001200	0.001201	0.001200
1.089582	0.558856	0.803287	0.100009	0.058381	0.001200	0.001202	0.001200

1.309293	0.527255	0.802120	0.100073	0.055609	0.001200	0.001201	0.001201
1.599910	0.537048	0.802057	0.100095	0.055133	0.001200	0.001200	0.001200
2.009687	0.558267	0.920256	0.100571	0.070281	0.001200	0.001201	0.001210
1.701977	0.581919	0.802021	0.100006	0.056582	0.001201	0.001209	0.001200
1.272031	0.546070	0.802306	0.100001	0.052283	0.001200	0.001203	0.001201
1.138254	0.571311	0.802120	0.100001	0.052604	0.001200	0.001205	0.001201
1.847854	0.544311	0.803650	0.100030	0.058433	0.001200	0.001205	0.001200
1.508705	0.559702	0.802014	0.100000	0.053608	0.001200	0.001205	0.001200
1.590177	0.529678	0.802296	0.100004	0.056836	0.001200	0.001200	0.001201
1.067240	0.526309	0.802056	0.100001	0.052999	0.001200	0.001201	0.001206
1.667128	0.633313	0.802044	0.100008	0.057859	0.001200	0.001207	0.001201
1.601226	0.565758	0.802057	0.100001	0.056022	0.001200	0.001205	0.001201
1.856149	0.577778	0.802419	0.100012	0.066226	0.001200	0.001200	0.001201
1.932781	0.568962	0.802220	0.100026	0.055878	0.001201	0.001214	0.001201
1.270729	0.546322	0.801965	0.100001	0.050552	0.001200	0.001203	0.001201
1.025200	0.503098	0.802042	0.100000	0.052326	0.001200	0.001205	0.001201
1.788584	0.570358	0.802220	0.100002	0.056178	0.001201	0.001201	0.001201
1.911098	0.548047	0.805260	0.100008	0.062874	0.001200	0.001211	0.001200
1.950299	0.573357	0.862519	0.100131	0.066241	0.001200	0.001231	0.001200
1.994273	0.551556	0.880969	0.100356	0.070892	0.001201	0.001201	0.001203
1.427852	0.566843	0.802010	0.100006	0.056670	0.001201	0.001209	0.001201
1.490485	0.610565	0.802043	0.100006	0.058994	0.001200	0.001207	0.001240
1.089582	0.533015	0.803287	0.100009	0.053814	0.001200	0.001202	0.001200
1.508705	0.559140	0.802014	0.100001	0.058222	0.001200	0.001200	0.001201
1.667128	0.566342	0.802044	0.100008	0.054933	0.001200	0.001207	0.001203
1.803505	0.571794	0.803586	0.100001	0.051906	0.001200	0.001204	0.001200
1.138254	0.577507	0.802120	0.100006	0.052677	0.001200	0.001205	0.001201

1.228591	0.543278	0.802131	0.100005	0.055046	0.001200	0.001200	0.001211
1.025107	0.531500	0.801962	0.100001	0.050500	0.001200	0.001205	0.001200
1.002959	0.559531	0.802338	0.100001	0.055743	0.001200	0.001200	0.001200
1.234679	0.553932	0.802568	0.100001	0.053587	0.001200	0.001201	0.001200
1.291660	0.514196	0.803391	0.100014	0.050266	0.001200	0.001205	0.001200
1.628020	0.535910	0.802057	0.100005	0.055133	0.001200	0.001201	0.001200
1.059534	0.511623	0.802235	0.100000	0.058533	0.001200	0.001200	0.001204
1.490485	0.610565	0.802043	0.100006	0.058994	0.001200	0.001207	0.001240
1.336732	0.591033	0.802010	0.100003	0.056670	0.001201	0.001209	0.001201
1.400414	0.527255	0.802120	0.100072	0.055609	0.001200	0.001201	0.001201
1.228591	0.543278	0.802131	0.100000	0.055046	0.001200	0.001200	0.001202
1.932781	0.568962	0.802220	0.100026	0.055910	0.001201	0.001214	0.001201
1.513017	0.514141	0.802014	0.100001	0.053649	0.001200	0.001200	0.001202
1.613172	0.537048	0.803188	0.100105	0.052897	0.001200	0.001200	0.001201

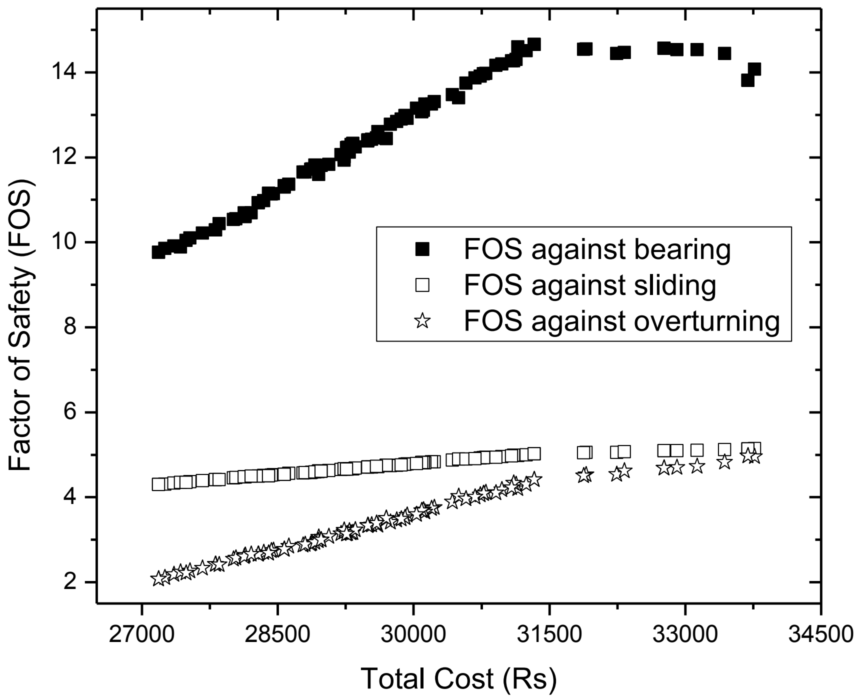

Fig. 11. Variation of factor of safety with total cost against bearing, sliding and overturning Figure for the RCC RW of height 3m and ϕ of 20^0

It can be seen that though there is a significant increase in the FOS against bearing with increase in total cost of the Retaining wall whereas the FOS against sliding shows marginal changes. It can also be concluded that FOS against sliding is the controlling factor for the considered retaining wall.

Table 5 The dimensions of the retaining wall and the percentage of reinforcement for the RW of 4m andϕ of 20^0

L_t	L_h	t	S	B	P_{ts}	P_{th}	P_{tt}
1.291207	1.545505	0.992268	0.207139	0.367227	0.001200	0.001202	0.001419
1.488601	1.323187	0.991964	0.182213	0.050318	0.001200	0.001202	0.001401
1.463549	1.160343	0.987039	0.182474	0.052259	0.001200	0.001202	0.001400
1.541281	1.349786	0.985759	0.182194	0.050141	0.001200	0.001203	0.001401
1.476706	1.652779	0.988834	0.182451	0.051876	0.001200	0.001248	0.001402
1.427810	1.711695	0.987041	0.182444	0.050393	0.001200	0.001200	0.001402
1.316216	1.597334	0.994586	0.182367	0.055599	0.001200	0.001200	0.001401
1.427810	1.715697	0.986842	0.182452	0.055475	0.001200	0.001223	0.001402
1.020082	1.142527	0.985760	0.182204	0.050082	0.001200	0.001203	0.001401
1.016088	1.556292	0.987395	0.182463	0.050286	0.001200	0.001208	0.001400
1.148976	1.020558	0.987919	0.182405	0.050684	0.001200	0.001209	0.001401
1.422851	1.741916	0.987295	0.182465	0.055400	0.001200	0.001225	0.001403
1.245422	1.000379	0.986200	0.182343	0.050098	0.001200	0.001203	0.001400
1.402516	1.147334	0.986469	0.182395	0.051995	0.001200	0.001200	0.001400
1.278385	1.715697	0.987041	0.182395	0.050245	0.001200	0.001224	0.001402
1.404054	1.494339	0.989417	0.182426	0.053782	0.001200	0.001201	0.001400
1.144910	1.550160	0.988107	0.182221	0.050286	0.001200	0.001200	0.001400
1.157562	1.739352	0.990325	0.182361	0.050703	0.001200	0.001200	0.001401
1.313114	1.125164	0.987349	0.182176	0.055875	0.001200	0.001202	0.001440
1.125217	1.467877	0.985880	0.182431	0.051612	0.001200	0.001200	0.001401
1.426453	1.205011	0.991666	0.182492	0.061871	0.001200	0.001204	0.001405
1.378958	1.655275	0.990857	0.182451	0.055910	0.001200	0.001248	0.001429
1.292508	1.125164	0.987349	0.182233	0.055052	0.001200	0.001210	0.001401
1.190449	1.726002	0.990616	0.182411	0.051103	0.001200	0.001200	0.001400

1.395666	1.748348	0.988207	0.182387	0.050104	0.001200	0.001200	0.001402
1.653548	1.174001	0.986472	0.182466	0.052981	0.001201	0.001200	0.001400
1.154289	1.474064	0.997498	0.182370	0.050557	0.001200	0.001203	0.001400
1.418367	1.015350	0.986225	0.182220	0.054836	0.001200	0.001206	0.001400
1.442624	1.160584	0.989206	0.182347	0.051474	0.001200	0.001202	0.001402
1.065857	1.334755	0.992712	0.181688	0.050755	0.001201	0.001202	0.001401
1.009458	1.556292	0.986283	0.182233	0.050286	0.001200	0.001208	0.001400
1.256468	1.681426	0.988414	0.182475	0.050245	0.001200	0.001224	0.001402
1.016238	1.174253	0.984988	0.182463	0.050312	0.001200	0.001212	0.001400
1.045434	1.554241	0.988333	0.182167	0.054356	0.001201	0.001200	0.001400
1.488601	1.362040	0.991964	0.182213	0.050318	0.001200	0.001202	0.001406
1.294791	1.716992	0.990673	0.182407	0.055114	0.001200	0.001208	0.001417
1.048045	1.007975	0.985925	0.182161	0.052896	0.001201	0.001200	0.001400
1.218894	1.164606	0.986200	0.182348	0.050098	0.001200	0.001206	0.001401
1.345521	1.554308	0.986589	0.182405	0.052929	0.001200	0.001203	0.001400
1.008434	1.622970	0.984373	0.182475	0.050219	0.001200	0.001201	0.001401
1.487890	1.733747	0.992260	0.182213	0.050318	0.001200	0.001200	0.001401
1.279853	1.409645	0.986215	0.182154	0.055490	0.001200	0.001202	0.001435
1.041363	1.841634	0.987025	0.182167	0.052896	0.001201	0.001207	0.001403
1.380603	1.412511	0.996274	0.182241	0.050854	0.001200	0.001202	0.001401
1.437672	1.569325	0.998107	0.182541	0.052368	0.001200	0.001203	0.001401
1.286800	1.773585	0.985905	0.182934	0.050770	0.001200	0.001207	0.001400
1.463549	1.727313	0.987039	0.182474	0.052259	0.001200	0.001202	0.001401
1.274108	1.702962	0.986215	0.182154	0.050142	0.001200	0.001220	0.001400
1.037060	1.167921	0.985925	0.182176	0.052896	0.001201	0.001200	0.001400
1.053591	1.597334	0.994402	0.181731	0.050925	0.001200	0.001200	0.001401
1.325457	1.173864	0.996194	0.182323	0.056751	0.001200	0.001201	0.001401

1.377477	1.241841	0.986250	0.182431	0.052127	0.001200	0.001201	0.001400
1.041016	1.090411	0.987349	0.182176	0.052751	0.001200	0.001200	0.001404
1.041016	1.050761	0.987349	0.182176	0.052751	0.001200	0.001201	0.001404
1.490532	1.349786	0.985780	0.182423	0.050035	0.001200	0.001203	0.001400
1.148976	1.031554	0.987919	0.182405	0.050812	0.001200	0.001209	0.001401
1.178563	1.135445	0.985760	0.182433	0.057002	0.001200	0.001209	0.001425
1.487983	1.506891	0.992288	0.182214	0.051859	0.001200	0.001202	0.001402
1.488508	1.733747	0.992513	0.182213	0.066726	0.001200	0.001245	0.001401
1.278385	1.745822	0.987041	0.182934	0.050245	0.001200	0.001224	0.001401
1.014385	1.460854	0.987328	0.182385	0.062328	0.001200	0.001200	0.001400
1.457847	1.220884	0.991916	0.182213	0.050771	0.001200	0.001202	0.001401
1.192341	1.591377	0.985828	0.182409	0.050554	0.001201	0.001202	0.001400
1.062832	1.174641	0.996010	0.181688	0.052078	0.001200	0.001201	0.001401
1.016007	1.089422	0.989730	0.182222	0.051586	0.001200	0.001201	0.001401

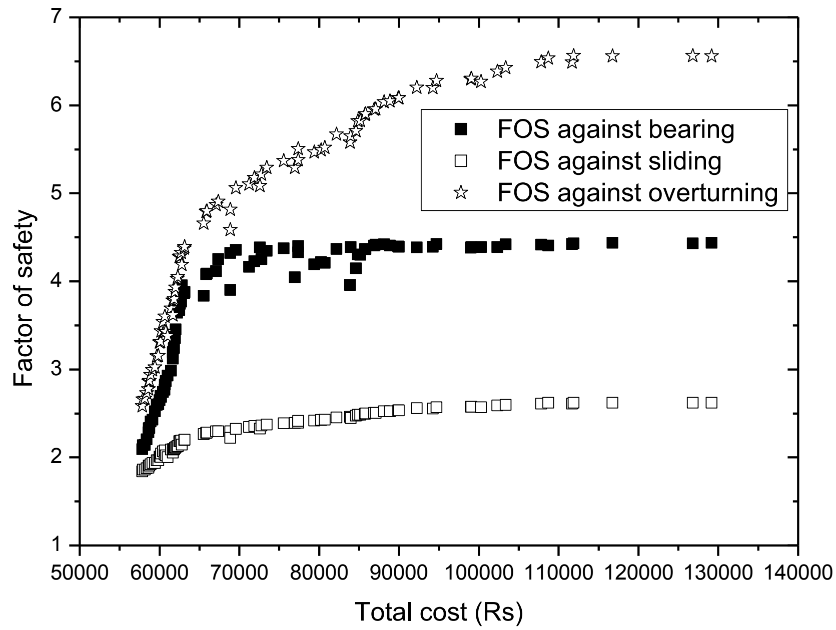

Fig. 12. Variations of factor of safety with total cost against bearing, sliding and overturning for height of 4.0m and angle of internal friction of 20^0

It can be seen that there is a steady increase in FOS against bearing to increase in cost for FOS>4.0, then after the FOS does not change appreciably with increase in cost. It was also found that FOS against sliding is the controlling factor for the considered retaining wall. Hence, there is an optimum cost option, where there is an increase in FOS, then after the changes in FOS is very marginal.

Table 6 The dimensions of the retaining wall and the percentage of reinforcement for the RW of 5m and ϕ of 20^0

L_t	L_h	T	S	B	P_{ts}	P_{th}	P_{tt}
1.068437	1.800890	0.777239	0.341946	0.100657	0.001202	0.001205	0.001201
1.081965	1.649108	0.779900	0.341572	0.101008	0.001202	0.001210	0.001201
1.837797	1.489164	0.781387	0.341907	0.109478	0.001200	0.001243	0.001212
1.101691	1.435661	0.779515	0.341910	0.100116	0.001201	0.001300	0.001206
1.249291	2.904240	0.779119	0.341756	0.100307	0.001201	0.001211	0.001202
1.410482	2.616071	0.780584	0.341787	0.104455	0.001200	0.001204	0.001236
1.225178	2.854711	0.781083	0.341907	0.100053	0.001201	0.001213	0.001206
1.374834	2.875661	0.784152	0.341997	0.101451	0.001202	0.001208	0.001225
1.190644	2.218469	0.778818	0.341906	0.100879	0.001201	0.001253	0.001258
1.626251	1.447109	0.778233	0.342045	0.101086	0.001200	0.001227	0.001203
1.070883	1.725280	0.780272	0.341736	0.100448	0.001201	0.001206	0.001295
1.178269	2.116140	0.780408	0.341915	0.100823	0.001202	0.001264	0.001203
1.143725	2.164726	0.780272	0.341736	0.100448	0.001201	0.001206	0.001315
1.349661	1.278381	0.778897	0.341752	0.101907	0.001204	0.001216	0.001204
1.784668	1.519569	0.781387	0.341907	0.117546	0.001200	0.001251	0.001213
1.012784	2.393307	0.780408	0.341921	0.101435	0.001202	0.001201	0.001201
1.168701	2.261519	0.778873	0.341906	0.100307	0.001200	0.001253	0.001224
1.926977	1.391865	0.779741	0.341245	0.100662	0.001206	0.001305	0.001206
1.108667	1.166907	0.778842	0.341938	0.101084	0.001201	0.001207	0.001203
1.426336	1.386489	0.779741	0.341245	0.100051	0.001206	0.001204	0.001221
1.065796	1.285791	0.779515	0.341879	0.100050	0.001201	0.001207	0.001207
1.231770	1.103266	0.778850	0.341871	0.100548	0.001201	0.001201	0.001207
1.033746	2.555226	0.781045	0.341469	0.100003	0.001202	0.001203	0.001201
1.178269	1.521821	0.780408	0.341915	0.100086	0.001200	0.001214	0.001203

1.033746	2.533713	0.781045	0.341469	0.101315	0.001201	0.001203	0.001202
1.030868	2.741961	0.781023	0.341494	0.101471	0.001202	0.001202	0.001201
1.187344	1.794469	0.785850	0.341881	0.102064	0.001201	0.001202	0.001206
1.054663	1.501897	0.779515	0.341879	0.100196	0.001201	0.001209	0.001200
1.162302	1.653384	0.780001	0.341880	0.100476	0.001201	0.001205	0.001201
1.219118	1.166907	0.781093	0.341907	0.100139	0.001200	0.001211	0.001207
1.138505	2.261519	0.778994	0.341906	0.100307	0.001200	0.001253	0.001207
1.130227	1.138290	0.778471	0.341907	0.100513	0.001201	0.001217	0.001206
1.837797	1.644184	0.781387	0.341907	0.107616	0.001200	0.001239	0.001212
1.155700	2.366604	0.778746	0.341809	0.101029	0.001201	0.001241	0.001202
1.168701	2.536328	0.778873	0.341906	0.100307	0.001200	0.001253	0.001254
1.233727	3.001296	0.781083	0.341921	0.100053	0.001201	0.001213	0.001205
1.275677	2.807233	0.778918	0.342595	0.100307	0.001201	0.001213	0.001202
1.343015	1.470923	0.779777	0.341514	0.100902	0.001207	0.001300	0.001200
1.142645	1.184334	0.779102	0.341907	0.100082	0.001201	0.001211	0.001201
1.228871	2.648249	0.779952	0.341888	0.100421	0.001202	0.001250	0.001330
1.294461	1.418803	0.779076	0.341750	0.100606	0.001202	0.001203	0.001239
1.245638	2.970194	0.778918	0.341835	0.101916	0.001201	0.001202	0.001203
1.202346	2.080960	0.780337	0.341644	0.100461	0.001202	0.001200	0.001298
1.307284	3.009613	0.792063	0.380623	0.100061	0.001201	0.001210	0.001651
1.249291	2.747082	0.779119	0.341756	0.100307	0.001202	0.001211	0.001203
1.030868	2.055477	0.780446	0.341514	0.101471	0.001202	0.001203	0.001202
1.065796	1.477918	0.779515	0.341879	0.100050	0.001201	0.001207	0.001207
1.296944	2.276738	0.780572	0.341873	0.101644	0.001204	0.001205	0.001210
1.766196	1.453034	0.778233	0.342039	0.101122	0.001200	0.001227	0.001203
1.074699	1.351279	0.779515	0.341879	0.100079	0.001201	0.001213	0.001206
1.233267	2.297992	0.780572	0.341702	0.100829	0.001204	0.001203	0.001210

1.718510	1.446563	0.778233	0.342045	0.101086	0.001200	0.001227	0.001206
1.045252	1.713056	0.778994	0.341878	0.100020	0.001201	0.001211	0.001206
1.006825	2.370660	0.783292	0.341878	0.100637	0.001200	0.001211	0.001206
1.125134	1.103266	0.783274	0.341878	0.100633	0.001201	0.001201	0.001207
1.251170	1.710832	0.779119	0.341756	0.100324	0.001201	0.001221	0.001202
1.142645	1.337045	0.779414	0.341999	0.101029	0.001201	0.001211	0.001201
1.174989	2.413817	0.780915	0.341907	0.101723	0.001201	0.001220	0.001209
1.248612	1.646379	0.778912	0.341932	0.101167	0.001201	0.001207	0.001200
1.626251	2.168028	0.778775	0.342045	0.101086	0.001200	0.001280	0.001203
1.174989	2.505694	0.779046	0.341907	0.101723	0.001201	0.001216	0.001202
1.303420	3.009613	0.780268	0.398614	0.100054	0.001202	0.001203	0.001448
1.088650	1.559336	0.780279	0.341984	0.100749	0.001200	0.001200	0.001204
1.180855	2.802624	0.780986	0.341895	0.101016	0.001200	0.001262	0.001202
1.169209	2.913141	0.780986	0.341895	0.101016	0.001202	0.001262	0.001202

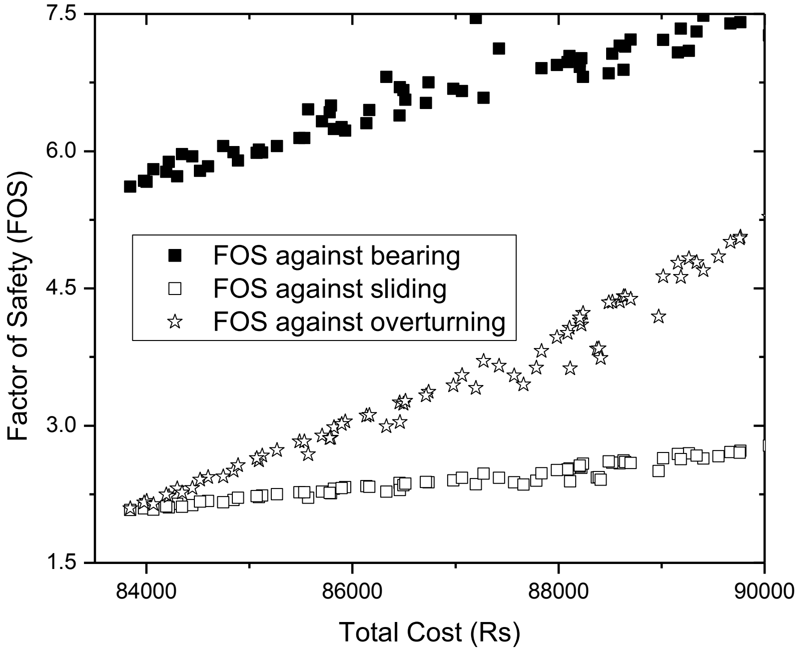

Fig. 13. Variation of factor of safety with total cost against bearing, sliding and overturning Figure for the RCC RW of height 5m and ϕ of 20^0

It can be seen that though the total cost for building a retaining wall of height 5m is different but the trend in variations of total cost with the FOS are similar. It can be seen that there is a steady increase in the FOS against overturning and bearing with increasing cost of the construction i.e increase in the dimension of the retaining wall.

Table 7 The dimensions of the retaining wall and the percentage of reinforcement for the RW of 6m and ϕ of 20^0

L_t	L_h	t	S	B	P_{ts}	P_{th}	P_{tt}
1.000943	4.999863	0.958437	0.924569	0.712614	0.001200	0.001223	0.001232
1.000071	4.999635	0.974494	1.021657	0.707845	0.001200	0.001209	0.001204
1.001013	4.999997	0.999104	1.208413	0.688922	0.001200	0.001229	0.001246
1.000453	4.999186	0.972416	0.472000	0.494812	0.001200	0.001201	0.001204
1.000060	4.999907	0.969015	0.472488	0.235983	0.001200	0.001202	0.001201
1.000472	4.999975	0.967891	0.472966	0.583429	0.001200	0.001200	0.001204
1.000158	4.999986	0.967891	0.472432	0.471749	0.001200	0.001202	0.001220
1.000702	4.999915	0.998011	1.480451	0.649066	0.001200	0.001204	0.001355
1.000272	4.999390	0.978498	0.471497	0.420179	0.001200	0.001203	0.001212
1.001028	4.999935	0.998556	1.357597	0.651905	0.001200	0.001214	0.001954
1.000703	4.999935	0.968679	0.969703	0.710020	0.001200	0.001201	0.001392
1.001011	5.000000	0.995680	1.234439	0.688922	0.001200	0.001229	0.001246
1.007906	4.999634	0.998079	0.798636	0.724501	0.001200	0.001223	0.001295
1.003515	4.999917	0.881555	0.651592	0.696868	0.001200	0.001220	0.001225
1.002052	4.999874	0.973163	0.471659	0.346502	0.001200	0.001202	0.001214
1.014251	4.996292	0.969221	0.472521	0.400622	0.001200	0.001212	0.001217
1.001028	4.999935	0.998556	1.352892	0.651905	0.001200	0.001214	0.001954
1.000712	4.999783	0.993323	1.435123	0.660040	0.001200	0.001200	0.001369
1.000043	4.999302	0.967789	0.472499	0.108612	0.001200	0.001201	0.001200
1.000453	4.999920	0.972416	0.471613	0.546234	0.001200	0.001203	0.001249
1.000820	4.999962	0.995366	0.494620	0.743033	0.001200	0.001209	0.001559
1.000060	4.999910	0.969015	0.472488	0.668592	0.001200	0.001202	0.001200

1.000379	4.999995	0.986486	0.666604	0.721444	0.001200	0.001204	0.001204
1.005250	4.999946	0.946072	0.688279	0.724007	0.001200	0.001212	0.001237
1.000105	4.999040	0.994945	1.036281	0.713919	0.001200	0.001204	0.001222
1.001668	4.999297	0.971880	0.471681	0.257575	0.001200	0.001203	0.001204
1.000121	4.999747	0.996653	0.890523	0.738432	0.001200	0.001201	0.001206
1.001155	4.999969	0.967789	0.472594	0.270757	0.001200	0.001201	0.001203
1.000184	4.999933	0.967847	0.472401	0.627153	0.001200	0.001201	0.001200
1.001267	4.999969	0.973183	0.472512	0.363413	0.001200	0.001202	0.001202
1.001051	4.999998	0.999583	1.176548	0.688922	0.001200	0.001444	0.001218
1.000204	4.997149	0.967582	0.519838	0.780537	0.001200	0.001207	0.001217
1.000325	4.999928	0.996334	1.383044	0.654387	0.001200	0.001206	0.001238
1.004141	4.999997	0.983813	0.785643	0.722550	0.001200	0.001200	0.001211
1.003576	4.998392	0.987336	1.235711	0.670262	0.001200	0.001211	0.001200
1.003954	4.999998	0.994190	1.169453	0.689914	0.001200	0.001202	0.001206
1.000655	4.999956	0.969744	0.472500	0.330276	0.001200	0.001202	0.001200
1.000043	4.998930	0.999735	0.469419	0.729729	0.001200	0.001201	0.001203
1.000018	4.999950	0.994884	0.542287	0.790472	0.001200	0.001231	0.001210
1.000116	4.999287	0.998571	0.467767	0.668523	0.001200	0.001201	0.001204
1.001568	4.999919	0.968334	0.472500	0.480030	0.001200	0.001202	0.001204
1.000171	4.999192	0.967789	0.472410	0.180464	0.001200	0.001200	0.001200
1.000200	4.999783	0.998498	1.460936	0.660040	0.001200	0.001212	0.001883
1.000369	4.999898	0.978450	0.885970	0.720868	0.001200	0.001206	0.001225
1.001720	4.999968	0.999327	1.105546	0.683973	0.001200	0.001202	0.001204
1.001480	4.999960	0.992702	1.148911	0.690144	0.001201	0.001208	0.001202
1.000453	4.999646	0.995743	1.077050	0.683815	0.001200	0.001211	0.001204
1.001480	4.999960	0.998697	1.148911	0.690144	0.001201	0.001208	0.001202
1.000429	4.999186	0.972416	0.472000	0.494812	0.001200	0.001201	0.001204

1.000178	4.999922	0.973657	0.977802	0.705524	0.001200	0.001204	0.001220
1.000379	4.999981	0.976798	0.698861	0.723765	0.001200	0.001211	0.001204
1.000013	4.999967	0.968130	0.472426	0.329515	0.001200	0.001202	0.001200
1.000200	4.999783	0.998498	1.460936	0.660040	0.001200	0.001212	0.001883
1.000043	4.999990	0.967789	0.472594	0.102972	0.001200	0.001201	0.001200
1.000269	4.999820	0.997663	0.813931	0.740633	0.001200	0.001225	0.001212
1.001720	4.999968	0.999327	1.124334	0.697976	0.001200	0.001202	0.001204
1.000157	4.999922	0.953396	0.977802	0.705524	0.001200	0.001204	0.001208
1.000013	4.999967	0.968130	0.472426	0.388435	0.001200	0.001202	0.001200
1.000354	4.999930	0.968334	0.472478	0.154469	0.001200	0.001202	0.001204
1.001720	4.999784	0.997597	1.128768	0.698741	0.001200	0.001202	0.001246
1.000444	4.999783	0.997597	1.123024	0.698741	0.001200	0.001202	0.001246
1.001720	4.999941	0.999327	1.124334	0.697976	0.001200	0.001202	0.001204
1.000006	4.999969	0.968379	0.472481	0.380955	0.001200	0.001202	0.001200
1.000143	4.999946	0.971869	0.472966	0.583429	0.001200	0.001200	0.001204
1.005250	4.999924	0.946072	0.702361	0.720052	0.001200	0.001212	0.001237

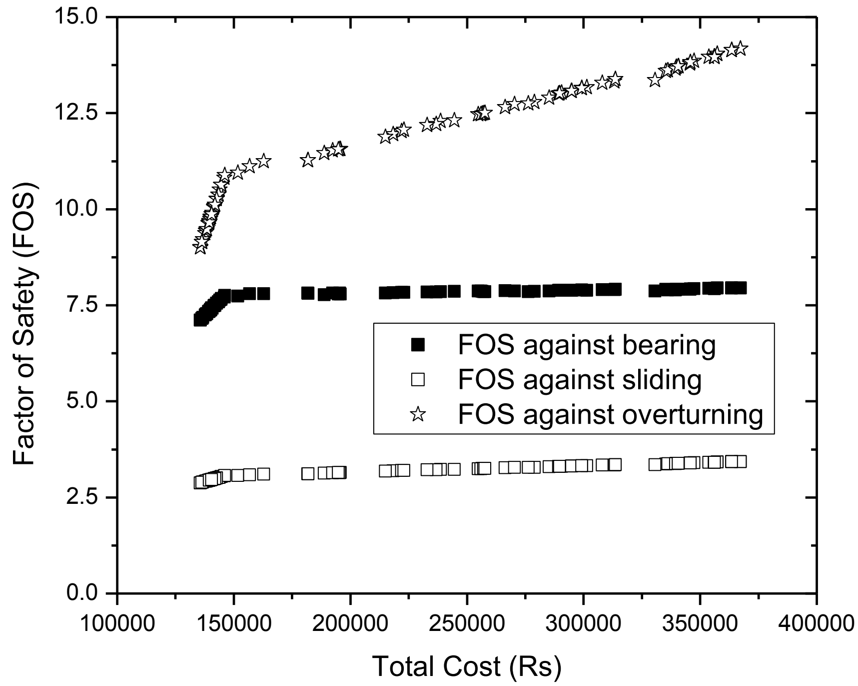

Fig. 14. Variation of factor of safety with total cost against bearing, sliding and overturning Figure for the RCC RW of height 6m and ϕ of 20^0

It is evident that due to in decrease in ϕ value, the total cost increased. It is also observed that, the increase in FOS against sliding is very less as compared to the FOS against bearing and overturning.

Table 8 The dimensions of the retaining wall and the percentage of reinforcement for the RW of 7m and ϕ of 20^0

L_t	L_h	t	S	B	P_{ts}	P_{th}	P_{tt}
1.883549	2.826453	0.991749	0.653579	0.101565	0.001200	0.001262	0.001204
2.724518	1.908309	0.999704	0.649582	0.108120	0.004540	0.006221	0.001266
2.890437	1.900427	0.997153	0.502223	0.104399	0.005327	0.007548	0.004870
2.871614	1.900679	0.973811	0.502223	0.101379	0.004165	0.001253	0.004870
2.517208	2.148268	0.994184	0.654817	0.101862	0.002432	0.001568	0.003554
2.247351	2.634007	0.984631	0.655261	0.102117	0.001200	0.001233	0.001239
2.742916	1.840739	0.942169	0.658667	0.103244	0.001650	0.001202	0.001209
3.014428	1.776613	0.996187	0.485006	0.100028	0.003643	0.002659	0.001769
1.543815	2.599614	0.989476	0.653528	0.101315	0.001200	0.001241	0.001202
2.250558	2.392585	0.981639	0.655385	0.100504	0.001200	0.001341	0.001231
1.551913	1.413553	0.991764	0.652005	0.100090	0.001200	0.001204	0.001201
2.681107	1.625418	0.998167	0.651592	0.108334	0.001200	0.001245	0.001201
1.657036	1.770498	0.992759	0.652844	0.100206	0.001200	0.001203	0.001213
1.653559	1.590260	0.988023	0.652844	0.100212	0.001200	0.001201	0.001216
2.571273	2.158902	0.999900	0.655705	0.103483	0.002763	0.001208	0.004126
2.968490	1.683329	0.990773	0.574165	0.102607	0.001766	0.001207	0.001275
2.968976	1.604528	0.995872	0.567651	0.100987	0.001766	0.001229	0.001756
1.526285	1.494612	0.991899	0.652807	0.101414	0.001200	0.001207	0.001218
1.820907	1.775813	0.981876	0.655223	0.100002	0.001200	0.001207	0.001203
2.724518	1.908309	0.999704	0.640369	0.108120	0.003942	0.006221	0.001250
2.578879	2.148268	0.970256	0.639916	0.105978	0.001886	0.001236	0.001229
2.723648	1.908309	0.999356	0.640369	0.110883	0.003942	0.006221	0.001250
2.571273	2.158902	0.999900	0.655649	0.103483	0.002763	0.001208	0.004126
1.586715	2.123870	0.982184	0.655142	0.101425	0.001200	0.001204	0.001205

2.136024	1.683382	0.979715	0.655223	0.101285	0.001200	0.001205	0.001200
1.624900	1.574350	0.988023	0.652844	0.100016	0.001200	0.001201	0.001228
2.733506	1.908309	0.999899	0.624764	0.114858	0.003496	0.005584	0.001273
1.960392	1.693114	0.991721	0.652686	0.101042	0.001200	0.001257	0.001201
3.014428	1.776613	0.996288	0.485006	0.100028	0.003643	0.002659	0.001769
2.905046	1.631478	0.991501	0.660778	0.105116	0.001544	0.001327	0.003785
2.305351	1.625418	0.998167	0.651114	0.107852	0.001200	0.001245	0.001201
1.577054	2.554409	0.991479	0.652005	0.100319	0.001200	0.001369	0.001201
2.186129	2.644198	0.979715	0.655275	0.101285	0.001200	0.001249	0.001200
2.724518	1.907397	0.985873	0.649837	0.103610	0.004540	0.006160	0.001212
2.733506	1.908309	0.999899	0.624764	0.115718	0.004540	0.004925	0.001267
2.103489	2.479744	0.984914	0.654216	0.101492	0.001200	0.001204	0.001206
1.966080	2.766232	0.991749	0.653590	0.101583	0.001200	0.001220	0.001213
1.948256	1.563248	0.995422	0.652005	0.100075	0.001200	0.001228	0.001202
2.545496	2.226513	0.997519	0.640502	0.100855	0.001354	0.001230	0.006173
2.890439	1.908309	0.998933	0.502223	0.107686	0.004735	0.006771	0.002053
2.710802	1.927983	0.999618	0.649400	0.108120	0.001886	0.001365	0.001211
2.196958	2.479493	0.984205	0.653528	0.102721	0.001201	0.001228	0.001206
2.733506	1.908309	0.999957	0.624764	0.115718	0.004540	0.004925	0.001267
2.197613	1.711894	0.980869	0.655385	0.103077	0.001200	0.001212	0.001208
1.847724	2.113309	0.992056	0.655636	0.100075	0.001200	0.001201	0.001201
1.921335	2.087780	0.991749	0.655636	0.100075	0.001200	0.001254	0.001202
2.256970	1.683382	0.979715	0.655223	0.101802	0.001200	0.001201	0.001207
1.421447	1.597749	0.989698	0.653505	0.100257	0.001200	0.001244	0.001202
1.847860	1.696476	0.991106	0.652686	0.100016	0.001200	0.001202	0.001204
2.136024	1.683382	0.979715	0.655223	0.101285	0.001200	0.001205	0.001200
1.653559	1.590260	0.999642	0.652844	0.100212	0.001200	0.001201	0.001216

2.948127	1.908155	0.999730	0.449612	0.100071	0.003377	0.002249	0.004581
2.890437	1.900427	0.997153	0.502223	0.104399	0.004370	0.007548	0.004870
2.358106	1.702530	0.991883	0.655143	0.104703	0.001201	0.001214	0.001227
2.297554	1.683382	0.979715	0.655223	0.101090	0.001200	0.001248	0.001206
2.491409	2.245677	0.970014	0.651291	0.100454	0.001253	0.001210	0.001205
2.578879	2.148268	0.970220	0.640400	0.105336	0.002183	0.001248	0.001699
1.835141	2.579738	0.988525	0.655578	0.100016	0.001200	0.001249	0.001204
1.577054	2.839497	0.995835	0.658014	0.100319	0.001200	0.001246	0.001201
1.891412	2.766232	0.991749	0.653590	0.101583	0.001200	0.001220	0.001213
2.145517	2.110494	0.997351	0.655518	0.101006	0.001200	0.001255	0.001202
2.851825	1.908148	0.999730	0.449612	0.100071	0.003370	0.002129	0.004581
1.624900	1.547917	0.987719	0.652844	0.100016	0.001200	0.001201	0.001205
1.703815	1.594496	0.992106	0.653528	0.101313	0.001200	0.001298	0.001207
2.145501	2.479493	0.984205	0.653520	0.102721	0.001201	0.001225	0.001206

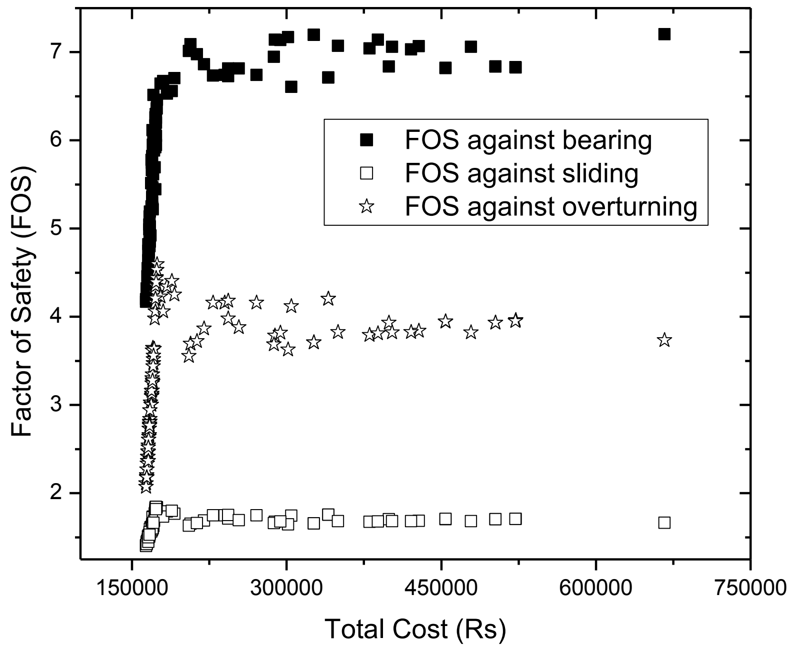

Fig. 15. Variation of factor of safety with total cost against bearing, sliding and overturning Figure for the RCC RW of height 7m and ϕ of 20^0

From the graph, it is observed that though there is very marginal increase in the FOS against sliding. But the peak increase in the FOS against overturning and bearing with increase in cost of Retaining wall.

Table 9 The dimensions of the retaining wall and the percentage of reinforcement for the RW of 8m and ϕ of 20^0

L_t	L_h	t	S	B	P_{ts}	P_{th}	P_{tt}
1.005042	4.988801	0.994265	1.262873	1.734585	0.001200	0.001848	0.001226
1.777406	4.902130	0.872573	0.885823	0.100038	0.001200	0.001868	0.001200
1.002117	4.994505	0.998798	1.164381	1.774534	0.001200	0.001507	0.001214
1.433419	4.952961	0.913344	0.876191	0.100074	0.001200	0.002154	0.001200
1.483422	4.998305	0.913344	0.876191	0.785784	0.001200	0.001930	0.001200
1.509832	4.999990	0.913344	0.876191	0.100078	0.001200	0.002090	0.001205
1.757785	4.999772	0.870380	0.885823	0.109492	0.001200	0.001777	0.001200
1.506165	4.998316	0.978332	0.876554	0.862446	0.001200	0.001512	0.001211
1.850073	4.998342	0.893928	0.886988	0.106336	0.001200	0.001807	0.001202
1.890975	4.999808	0.921564	0.885399	0.219100	0.001200	0.001759	0.001212
1.455976	4.999996	0.913344	0.876191	0.100307	0.001200	0.002157	0.001205
1.768754	4.964555	0.869757	0.885868	0.100039	0.001200	0.001877	0.001200
1.028544	4.949422	0.848459	0.891043	1.723743	0.001200	0.001221	0.001201
1.509832	4.999990	0.913344	0.876191	0.100078	0.001200	0.002090	0.001205
1.970475	4.999717	0.913344	0.886988	0.107204	0.001200	0.001323	0.001373
1.004971	4.991781	0.999832	0.950763	1.823854	0.001200	0.001227	0.001222
1.755717	4.994365	0.980322	0.861450	0.475253	0.001201	0.001213	0.001211
1.433419	4.998316	0.919453	0.877863	0.862446	0.001200	0.001284	0.001200
1.006729	4.998600	0.999487	0.875891	1.802141	0.001200	0.001286	0.001200
1.002950	4.999655	0.859322	0.891043	1.619135	0.001200	0.001221	0.001652
1.651079	4.976605	0.913114	0.876191	0.100005	0.001200	0.001908	0.001200
1.011029	4.998981	0.993760	1.387609	1.687265	0.001200	0.001236	0.001223
1.001216	4.957623	0.849284	0.891043	1.619135	0.001200	0.001221	0.001203
1.725671	4.981393	0.912712	0.876250	0.100071	0.001201	0.001841	0.001200

1.005512	4.999980	0.993470	1.144294	1.775191	0.001200	0.001830	0.001211
1.678975	4.973095	0.913114	0.876191	0.100071	0.001200	0.001841	0.001248
1.501170	4.998316	0.919453	0.877875	0.862446	0.001200	0.001284	0.001200
1.005937	4.999980	0.993470	1.144294	1.775191	0.001200	0.001830	0.001214
1.010953	4.998605	0.999772	1.392986	1.691672	0.001200	0.001236	0.001212
1.626087	4.997841	0.979739	0.862970	0.682122	0.001201	0.001205	0.001200
1.501170	4.998316	0.933461	0.877875	0.892420	0.001200	0.001235	0.001230
1.004674	4.998728	0.997716	1.001306	1.816983	0.001200	0.001203	0.001726
1.004610	4.998719	0.999668	0.903101	1.816922	0.001200	0.001212	0.001397
1.002117	4.989698	0.998798	1.180031	1.774534	0.001200	0.001338	0.001205
1.122985	4.991960	0.853289	0.890621	1.466110	0.001200	0.001217	0.001205
1.010953	4.999968	0.999733	1.407877	1.688687	0.001200	0.001444	0.001218
1.011031	4.998605	0.999858	1.328371	1.693976	0.001200	0.001236	0.001206
1.000400	4.999398	0.999228	1.284213	1.736232	0.001200	0.001205	0.001203
1.014605	4.998330	0.933900	0.981347	1.781070	0.001200	0.001204	0.001217
1.001399	4.999692	0.995524	1.212581	1.760394	0.001200	0.001218	0.001208
1.004971	4.998718	0.999886	0.906327	1.847256	0.001200	0.001219	0.001533
1.002259	4.988242	0.998822	0.864784	1.779212	0.001200	0.001211	0.001204
1.004166	4.999165	0.996097	1.112486	1.742889	0.001200	0.001308	0.001203
1.295094	4.997193	0.934438	0.897397	1.263078	0.001200	0.001215	0.001224
1.586547	4.972394	0.913114	0.876191	0.100102	0.001200	0.001867	0.001200
1.295094	4.985662	0.832258	0.897407	1.263078	0.001200	0.001215	0.001224
1.313722	4.998598	0.934438	0.876403	1.263078	0.001200	0.001388	0.001207
1.433419	4.980974	0.913344	0.876191	0.100074	0.001200	0.002154	0.001200
1.002117	4.994505	0.998798	1.180031	1.774534	0.001200	0.001507	0.001203
1.433419	4.989274	0.919453	0.877574	0.862446	0.001200	0.001215	0.001200
1.003625	4.997463	0.999867	1.115255	1.779212	0.001200	0.001290	0.001204

1.295094	4.985662	0.832258	0.897407	1.242361	0.001200	0.001215	0.001224
1.749810	4.998896	0.980173	0.877026	0.517426	0.001201	0.001225	0.001211
1.003099	4.999948	0.997928	1.061328	1.807717	0.001200	0.001221	0.001665
1.005327	4.999343	0.998885	1.286779	1.736232	0.001200	0.001205	0.001203
1.002861	4.999145	0.998111	1.425797	1.681240	0.001200	0.001512	0.001273
1.002199	4.998902	0.995952	0.990667	1.807447	0.001200	0.001205	0.001399
1.001399	4.999692	0.995524	1.350462	1.724102	0.001200	0.001217	0.001208
1.003119	4.998730	0.995981	0.982124	1.807465	0.001200	0.001202	0.001450
1.626087	4.997066	0.979739	0.861450	0.629630	0.001201	0.001214	0.001201
1.005512	4.999980	0.993470	1.122965	1.775191	0.001200	0.001830	0.001211
1.000897	4.999821	0.997928	1.043071	1.807770	0.001200	0.001232	0.001740
1.480433	4.995182	0.906985	0.887275	0.954094	0.001200	0.001218	0.001232
1.302079	4.993473	0.848554	0.890895	1.161579	0.001200	0.001205	0.001202
1.031055	4.998660	0.999886	0.879683	1.805459	0.001200	0.001204	0.001206

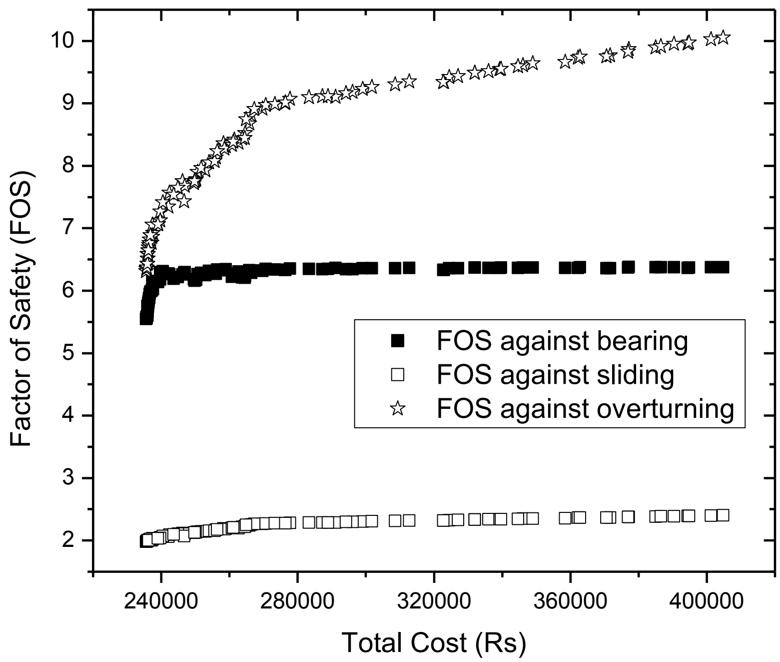

Fig. 16. Variation of factor of safety with total cost against bearing, sliding and overturning Figure for the RCC RW of height 8m and ϕ of 20^0

It can be seen that there is a steady increase in FOS against bearing to increase in cost for FOS>6.0, then after the FOS does not change appreciably with increase in cost. It was also found that FOS against sliding is the controlling factor for the considered retaining wall. Hence, there is an optimum cost option, where there is an increase in FOS, then after the changes in FOS is very marginal.

Table 10 The dimensions of the retaining wall and the percentage of reinforcement for the RW of 8m and ϕ of 40^0

L_t	L_h	t	S	B	P_{ts}	P_{th}	P_{tt}
1.223192	2.210834	0.900377	0.458557	0.100390	0.001200	0.001207	0.001224
1.049841	1.911125	0.907437	0.457086	0.100013	0.001200	0.001200	0.001212
1.154045	2.809181	0.895618	0.458250	0.100403	0.001200	0.001200	0.001392
1.298829	1.210323	0.894675	0.458130	0.100079	0.001200	0.001200	0.001207
1.091489	2.693822	0.903880	0.458153	0.100320	0.001200	0.001200	0.001258
1.082825	4.994827	0.898573	0.508176	0.102453	0.001201	0.001226	0.001430
1.087682	4.977192	0.930188	0.594814	0.102036	0.001201	0.001205	0.001320
1.042744	3.955944	0.894229	0.458099	0.100088	0.001201	0.001201	0.001205
1.121685	4.484193	0.898593	0.458429	0.100904	0.001201	0.001202	0.001247
1.087682	4.980332	0.924109	0.589608	0.102252	0.001201	0.001205	0.001350
1.104436	4.789871	0.895203	0.459658	0.100682	0.001200	0.001201	0.001200
1.051197	2.613908	0.895618	0.458128	0.100044	0.001200	0.001200	0.001208
1.119135	4.438920	0.898593	0.458429	0.100804	0.001200	0.001202	0.001249
1.016975	3.628177	0.898254	0.458131	0.100774	0.001201	0.001202	0.001200
1.221915	2.475013	0.906642	0.458297	0.100102	0.001200	0.001207	0.001203
1.136496	4.122251	0.906149	0.458098	0.100763	0.001200	0.001201	0.001215
1.086127	4.226290	0.897947	0.457925	0.100538	0.001200	0.001200	0.001204
1.081033	2.208098	0.898011	0.458349	0.101463	0.001200	0.001200	0.001204
1.087682	4.988162	0.880100	0.464666	0.102366	0.001200	0.001201	0.001203
1.095098	3.282224	0.898470	0.458153	0.100771	0.001200	0.001202	0.001200
1.118178	4.729057	0.896050	0.458186	0.101290	0.001200	0.001201	0.001201
1.088931	3.151468	0.898462	0.458429	0.100804	0.001200	0.001202	0.001249
1.153075	3.272823	0.900641	0.458250	0.100272	0.001200	0.001200	0.001389
1.012143	1.466913	0.894868	0.458557	0.100070	0.001200	0.001201	0.001206

1.037197	3.055570	0.894750	0.458107	0.101448	0.001200	0.001201	0.001212
1.031977	3.003283	0.899048	0.458435	0.100063	0.001200	0.001202	0.001250
1.086437	4.998160	0.898899	0.538978	0.100115	0.001200	0.001207	0.001214
1.118178	4.675412	0.896050	0.458186	0.101290	0.001200	0.001201	0.001201
1.214537	1.059149	0.901519	0.458090	0.100394	0.001200	0.001207	0.001200
1.004760	3.849629	0.894637	0.458094	0.100087	0.001201	0.001202	0.001240
1.030102	4.943352	0.886029	0.459487	0.100372	0.001200	0.001200	0.001204
1.223192	2.130999	0.898005	0.457910	0.100390	0.001200	0.001201	0.001207
1.103386	4.106958	0.897766	0.458242	0.101380	0.001200	0.001206	0.001219
1.075851	4.991184	0.982845	0.631975	0.102248	0.001200	0.001202	0.001214
1.033345	2.809181	0.895618	0.458250	0.100389	0.001200	0.001200	0.001392
1.112715	1.172040	0.895730	0.458088	0.100365	0.001200	0.001201	0.001200
1.123167	1.310307	0.894683	0.458130	0.100123	0.001200	0.001200	0.001207
1.087682	4.992684	0.926907	0.544867	0.102318	0.001200	0.001204	0.001441
1.089894	2.971799	0.898492	0.458248	0.102497	0.001200	0.001200	0.001227
1.087682	4.988162	0.969147	0.464666	0.102366	0.001200	0.001204	0.001205
1.000013	1.911125	0.907437	0.457086	0.100013	0.001200	0.001200	0.001212
1.089535	4.998160	0.898899	0.517509	0.100115	0.001200	0.001207	0.001253
1.087668	4.990397	0.928030	0.602857	0.102036	0.001201	0.001205	0.001338
1.047470	2.387779	0.900377	0.458557	0.100390	0.001200	0.001207	0.001257
1.089702	4.996618	0.969147	0.471568	0.102235	0.001200	0.001207	0.001206
1.123167	1.257152	0.894683	0.458130	0.100123	0.001200	0.001200	0.001207
1.159195	3.457388	0.897971	0.458248	0.101390	0.001200	0.001200	0.001229
1.001083	3.480952	0.894206	0.458088	0.100677	0.001201	0.001202	0.001211
1.101355	2.875675	0.903076	0.458441	0.100146	0.001200	0.001205	0.001398
1.115084	3.983769	0.894229	0.458099	0.100088	0.001201	0.001202	0.001206
1.010398	1.733094	0.895975	0.458163	0.100194	0.001200	0.001208	0.001202

1.087682	4.986121	0.924109	0.569244	0.102252	0.001200	0.001201	0.001229
1.087668	4.988066	0.884139	0.461363	0.102070	0.001201	0.001205	0.001205
1.035112	1.880234	0.894750	0.458107	0.100195	0.001200	0.001201	0.001212
1.054879	3.222471	0.907767	0.457214	0.102234	0.001200	0.001200	0.001212
1.042744	3.933540	0.894229	0.458099	0.100677	0.001201	0.001200	0.001211
1.221915	1.572163	0.904697	0.458156	0.100102	0.001200	0.001200	0.001203
1.298829	2.312147	0.894675	0.458271	0.100131	0.001200	0.001207	0.001206
1.104009	1.003593	0.895209	0.458090	0.100394	0.001200	0.001200	0.001200
1.053866	2.091737	0.922435	0.457214	0.100025	0.001200	0.001200	0.001212
1.087129	4.991184	0.982845	0.613413	0.102393	0.001200	0.001202	0.001214
1.214537	1.350055	0.901598	0.458090	0.100394	0.001200	0.001205	0.001254
1.089969	3.676555	0.912534	0.456844	0.100337	0.001200	0.001200	0.001210
1.196894	3.342192	0.897499	0.458228	0.101383	0.001200	0.001206	0.001381
1.071157	4.995504	0.909016	0.644881	0.102460	0.001200	0.001226	0.001430

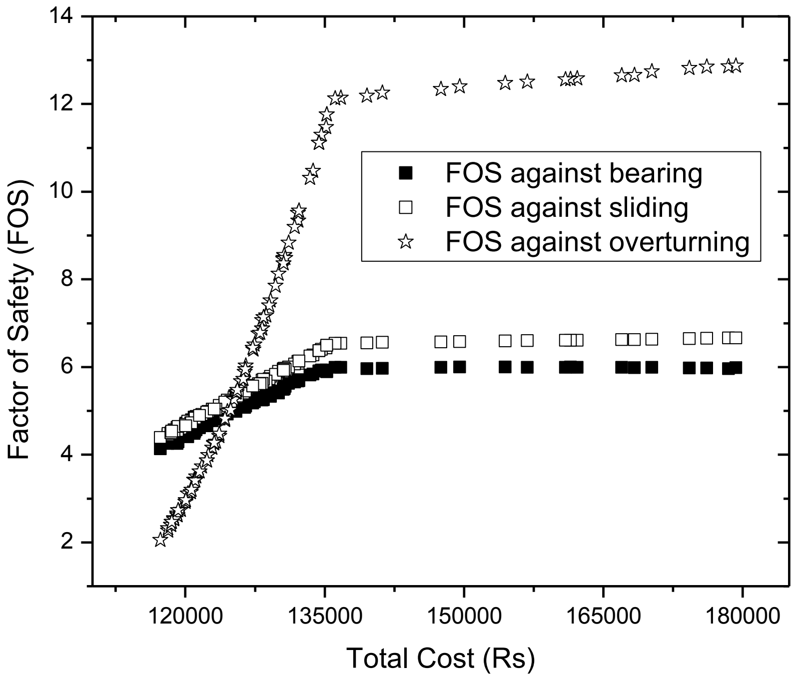

Fig. 17. Variation of factor of safety with total cost against bearing, sliding and overturning Figure for the RCC RW of height 8m and ϕ of 40^0

From the above graph, it can be seen that there is a very steep increase in the FOS against overturning with increase in the cost of RW. At the same time there is marginal increase in the FOS against bearing and sliding. For this case, the FOS against bearing is the controlling factor.

Table 11 The dimensions of the retaining wall and the percentage of reinforcement for the RW of 9m and ϕ of 20^0

L_t	L_h	t	S	B	P_{ts}	P_{th}	P_{tt}
1.828089	3.575005	0.928619	1.105470	0.100755	0.001200	0.002195	0.001209
2.014382	4.994547	0.950118	1.110389	0.214957	0.001200	0.001841	0.001343
1.118460	4.992050	0.981622	1.091868	1.279084	0.001200	0.001690	0.001213
2.126716	2.641315	0.959327	1.097288	0.102475	0.001200	0.002158	0.001222
2.061682	1.694547	0.964338	1.098287	0.100172	0.001200	0.002008	0.001257
2.010205	1.405781	0.951750	1.098328	0.106895	0.001200	0.002677	0.001215
2.053869	4.688210	0.951490	1.098286	0.102245	0.001200	0.001878	0.001293
2.135398	1.569111	0.960299	1.097814	0.101074	0.001200	0.001880	0.001208
2.068933	3.386599	0.965092	1.096844	0.100334	0.001200	0.001652	0.001227
2.049040	4.168330	0.950056	1.098292	0.101550	0.001200	0.001821	0.001208
1.105608	4.997884	0.998328	1.365147	1.999700	0.001200	0.001362	0.001201
2.176003	3.713577	0.965092	1.096844	0.101111	0.001200	0.001842	0.001248
2.061682	2.047017	0.964338	1.098287	0.102272	0.001200	0.002008	0.001257
2.128850	3.201239	0.964064	1.096937	0.102918	0.001200	0.001842	0.001207
1.991381	4.999393	0.965445	1.100506	0.440334	0.001200	0.002244	0.001202
1.123250	4.997484	0.991705	1.091941	1.568319	0.001200	0.001366	0.001201
2.020499	4.404338	0.953618	1.099879	0.102894	0.001200	0.001878	0.001276
2.053140	2.843739	0.951355	1.098362	0.102043	0.001200	0.001818	0.001252
1.118460	4.992050	0.990579	1.091868	1.279084	0.001200	0.001690	0.001213
2.024163	1.598211	0.950809	1.099013	0.100685	0.001200	0.002233	0.001228
1.147683	4.996944	0.996568	1.225045	1.999991	0.001200	0.001389	0.001587
2.035043	3.634044	0.951376	1.098321	0.100199	0.001200	0.001825	0.001211
2.010205	1.405781	0.951732	1.098269	0.102274	0.001200	0.002677	0.001200

2.135398	3.777547	0.964060	1.098371	0.102958	0.001200	0.001818	0.001263
2.145583	2.916404	0.964060	1.098120	0.101090	0.001200	0.001702	0.001263
1.165758	4.999988	0.960588	1.149510	1.999980	0.001200	0.001802	0.001259
2.125418	3.112579	0.963816	1.095682	0.104943	0.001200	0.001842	0.001245
2.238236	4.399834	0.953618	1.099879	0.100471	0.001200	0.002114	0.001469
2.035790	4.602122	0.953228	1.098286	0.102215	0.001200	0.001878	0.001273
1.148214	4.997495	0.962902	1.097812	1.670363	0.001200	0.001509	0.001211
1.153208	4.996733	0.996568	1.229784	1.999991	0.001200	0.001317	0.001414
2.115736	3.476357	0.952079	1.098395	0.100291	0.001200	0.002173	0.001211
2.166419	3.234655	0.964906	1.097809	0.112284	0.001200	0.001822	0.001211
2.133513	1.823166	0.951621	1.098374	0.101997	0.001200	0.002165	0.001205
2.077217	4.994547	0.959185	1.113122	0.220841	0.001200	0.002011	0.001344
2.154051	2.258575	0.950702	1.099052	0.111305	0.001200	0.001973	0.001202
2.037923	2.370685	0.956186	1.098284	0.102181	0.001200	0.002351	0.001247
1.105608	4.997884	0.996910	1.384575	1.999700	0.001200	0.001362	0.001202
2.125418	3.112579	0.963816	1.095567	0.137200	0.001200	0.001542	0.001245
1.146209	4.999446	0.990579	1.091868	1.910004	0.001200	0.001690	0.001213
1.123250	4.997495	0.995677	1.087792	1.542109	0.001200	0.001372	0.001202
1.147683	4.999928	0.996568	1.173909	1.999991	0.001200	0.001389	0.001587
1.976754	4.997766	0.970510	1.095082	0.623542	0.001200	0.001723	0.001240
1.179120	4.998037	0.984814	1.184728	1.999997	0.001200	0.001882	0.001231
2.073475	4.128323	0.965904	1.096395	0.101530	0.001200	0.001756	0.001210
2.079131	4.994547	0.960511	1.112542	0.334622	0.001200	0.001991	0.001345
2.059433	2.748135	0.964976	1.096844	0.100477	0.001200	0.001842	0.001245
1.107895	4.997592	0.994408	1.439495	1.999220	0.001200	0.001429	0.001699
1.153315	4.996799	0.996568	1.284868	1.999991	0.001200	0.001324	0.001414
2.009487	1.531395	0.949390	1.099005	0.102512	0.001200	0.002186	0.001251

2.022995	1.886383	0.950701	1.099013	0.100685	0.001200	0.002234	0.001234
1.183329	4.999694	0.999713	1.113630	1.999920	0.001200	0.001783	0.001269
1.123250	4.997495	0.999168	1.087792	1.969950	0.001200	0.001381	0.001645
2.008319	1.819568	0.949281	1.099005	0.102512	0.001200	0.002187	0.001235
2.221869	4.399834	0.953618	1.099879	0.244753	0.001200	0.001688	0.001443
2.058568	4.989558	0.965401	1.100448	0.214957	0.001200	0.001790	0.001217
1.113616	4.996068	0.969104	1.428968	1.999711	0.001200	0.001489	0.001269
2.073475	4.223581	0.972479	1.098273	0.101530	0.001200	0.001756	0.001210
1.105608	4.997884	0.997746	1.336320	1.999700	0.001200	0.001362	0.001202
2.001229	2.406202	0.951194	1.099748	0.101894	0.001200	0.002162	0.001200
1.143566	4.997495	0.996204	1.095853	1.749486	0.001200	0.001389	0.001208
2.001229	2.000236	0.949652	1.098985	0.101199	0.001200	0.002233	0.001224
2.133556	1.459009	0.949347	1.098421	0.100692	0.001200	0.002107	0.001200
2.063855	1.520118	0.951990	1.098974	0.102144	0.001200	0.002186	0.001205
1.153208	4.996086	0.996568	1.274787	1.999715	0.001200	0.001324	0.001250

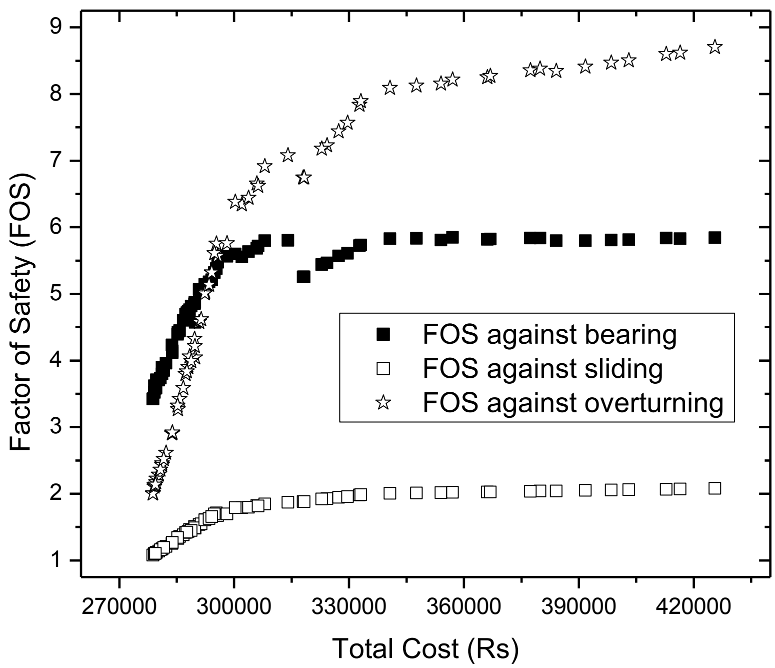

Fig. 18. Variation of factor of safety with total cost against bearing, sliding and overturning Figure for the RCC RW of height 9m and ϕ of 20^0

Table 12 The dimensions of the retaining wall and the percentage of reinforcement for the RW of 9m and ϕ of 40^0

L_t	L_h	T	S	b	P_{ts}	P_{th}	P_{tt}
1.000003	2.390834	0.208391	0.100020	0.100319	0.001200	0.001217	0.013455
1.336551	4.023791	0.275883	0.100038	0.100801	0.001200	0.001201	0.017612
1.019715	3.089197	0.225642	0.100007	0.100199	0.001200	0.001220	0.013397
1.019715	3.121603	0.225642	0.100007	0.100005	0.001200	0.001214	0.013397
1.355252	4.008653	0.275617	0.100085	0.100418	0.001200	0.001261	0.018509
1.019848	2.446468	0.217987	0.100004	0.100028	0.001200	0.001200	0.012404
1.019715	3.089197	0.226235	0.100007	0.100178	0.001200	0.001220	0.013353
1.046189	2.000817	0.201600	0.100014	0.100014	0.001200	0.001206	0.013903
1.019848	2.420000	0.217987	0.100004	0.100036	0.001200	0.001200	0.012404
1.010125	2.635406	0.226420	0.100011	0.100009	0.001200	0.001200	0.011306
1.001932	3.785636	0.234708	0.100014	0.100020	0.001200	0.001210	0.012630
1.006932	2.818134	0.225584	0.100000	0.100035	0.001200	0.001212	0.012045
1.000507	2.764249	0.225190	0.100006	0.100031	0.001200	0.001215	0.011952
1.017088	3.915811	0.237354	0.100007	0.100010	0.001200	0.001214	0.012568
1.021715	2.591874	0.223473	0.100006	0.100033	0.001200	0.001208	0.012210
1.046189	2.000817	0.201737	0.100017	0.100014	0.001200	0.001206	0.013903
1.000007	2.359755	0.208391	0.100003	0.100020	0.001200	0.001201	0.013362
1.512937	3.599663	0.290635	0.100017	0.100266	0.001200	0.001202	0.020441
1.041761	2.598369	0.226134	0.100006	0.100034	0.001200	0.001211	0.012303
1.355252	4.008653	0.275617	0.100033	0.100418	0.001200	0.001261	0.018509
1.000007	2.359755	0.208391	0.100003	0.100020	0.001200	0.001201	0.013362
1.000507	2.766123	0.226175	0.100004	0.100036	0.001200	0.001216	0.012360
1.019848	2.420000	0.217987	0.100003	0.100036	0.001200	0.001200	0.012404
1.001932	3.785636	0.234708	0.100014	0.100020	0.001200	0.001210	0.012630

1.019715	3.089197	0.225642	0.100007	0.100199	0.001200	0.001220	0.013397
1.022274	2.734846	0.225190	0.100006	0.100032	0.001200	0.001215	0.012352
1.019715	3.089197	0.226235	0.100007	0.100182	0.001200	0.001220	0.013353
1.011982	3.757664	0.234708	0.100013	0.100031	0.001200	0.001207	0.012630
1.002357	3.504827	0.236669	0.100001	0.100209	0.001200	0.001210	0.012578
1.011982	3.757664	0.234708	0.100013	0.100031	0.001200	0.001207	0.012630
1.011982	3.757664	0.234708	0.100013	0.100031	0.001200	0.001207	0.012630
1.027854	2.205348	0.208211	0.100027	0.100014	0.001200	0.001201	0.013900
1.004060	2.905345	0.227359	0.100016	0.100050	0.001200	0.001200	0.012048
1.022787	2.805673	0.226510	0.100001	0.100035	0.001200	0.001207	0.012424
1.008169	4.101389	0.240407	0.100023	0.100215	0.001200	0.001206	0.012517
1.010125	2.666478	0.226455	0.100011	0.100036	0.001200	0.001201	0.011416
1.355252	4.022090	0.275617	0.100085	0.100418	0.001200	0.001261	0.018509
1.010765	2.818134	0.225584	0.100000	0.100005	0.001200	0.001202	0.012791
1.071526	4.093202	0.243699	0.100033	0.100022	0.001200	0.001203	0.013923
1.426147	3.802664	0.281190	0.100050	0.100910	0.001200	0.001202	0.019482
1.019715	3.630757	0.210869	0.100043	0.100955	0.001200	0.001220	0.019953
1.001932	3.785636	0.234708	0.100014	0.100020	0.001200	0.001210	0.012630
1.355252	4.022090	0.275617	0.100085	0.100418	0.001200	0.001261	0.018509
1.001932	3.785636	0.234708	0.100014	0.100020	0.001200	0.001210	0.012630
1.426147	3.960351	0.295619	0.100047	0.100910	0.001200	0.001205	0.019482
1.021715	2.695952	0.225135	0.100001	0.100033	0.001200	0.001222	0.012210
1.028787	4.581210	0.237656	0.100021	0.100027	0.001200	0.001220	0.014017
1.512937	3.599663	0.294930	0.100020	0.102245	0.001200	0.001204	0.018834
1.013220	2.734846	0.225111	0.100006	0.100032	0.001200	0.001208	0.012352
1.006169	3.927036	0.226455	0.100032	0.100030	0.001200	0.001205	0.014572
1.006974	3.944596	0.237983	0.100003	0.100031	0.001200	0.001210	0.012382

1.011982	3.757664	0.234708	0.100013	0.100031	0.001200	0.001207	0.012630
1.022787	2.805673	0.226510	0.100002	0.100035	0.001200	0.001207	0.012424
1.003828	2.614560	0.226420	0.100002	0.100013	0.001200	0.001201	0.011238
1.027854	2.205348	0.208211	0.100027	0.100014	0.001200	0.001201	0.013900
1.356745	4.022090	0.277255	0.100049	0.100422	0.001200	0.001205	0.018640
1.005167	2.656682	0.225874	0.100004	0.100031	0.001200	0.001204	0.011563
1.016597	2.740745	0.226355	0.100006	0.100002	0.001200	0.001203	0.011970
1.001932	3.785636	0.234708	0.100014	0.100020	0.001200	0.001210	0.012630
1.019848	2.420000	0.217987	0.100003	0.100036	0.001200	0.001200	0.012404
1.002251	2.438508	0.225642	0.100000	0.100043	0.001200	0.001217	0.010748
1.004060	2.905545	0.227359	0.100016	0.100050	0.001200	0.001200	0.012048
1.011982	3.757664	0.234708	0.100013	0.100031	0.001200	0.001207	0.012630
1.022324	4.023791	0.219718	0.100010	0.100801	0.001200	0.001201	0.017612
1.001332	3.089197	0.226451	0.100000	0.100016	0.001200	0.001212	0.012813
1.000507	2.766123	0.226175	0.100004	0.100036	0.001200	0.001216	0.012360
1.010125	2.559111	0.226420	0.100011	0.100036	0.001200	0.001200	0.011306
1.008008	2.924997	0.227591	0.100016	0.100049	0.001200	0.001206	0.012085
1.000027	2.379019	0.208498	0.100003	0.100042	0.001200	0.001201	0.013362
1.019696	3.069932	0.225534	0.100008	0.100331	0.001200	0.001220	0.013397
1.001348	3.102437	0.226451	0.100002	0.100016	0.001200	0.001207	0.012636
1.046189	2.000817	0.201737	0.100032	0.100014	0.001200	0.001208	0.013903
1.022274	2.894864	0.227671	0.100007	0.100036	0.001200	0.001209	0.012360
1.038839	2.588505	0.225408	0.100006	0.100028	0.001200	0.001200	0.012385
1.010125	2.559111	0.226420	0.100011	0.100036	0.001200	0.001200	0.011306
1.001932	3.785636	0.234708	0.100014	0.100020	0.001200	0.001210	0.012630
1.071526	4.090849	0.243699	0.100033	0.100022	0.001200	0.001203	0.013923
1.002378	3.780948	0.234708	0.100014	0.100023	0.001200	0.001210	0.012630

1.005723	3.931724	0.226455	0.100018	0.100034	0.001200	0.001207	0.014572
1.038847	2.589565	0.226134	0.100006	0.100034	0.001200	0.001211	0.012303
1.041754	2.597309	0.225408	0.100002	0.100028	0.001200	0.001200	0.012385
1.010125	2.559111	0.222643	0.100011	0.100036	0.001200	0.001200	0.012023
1.004194	2.959261	0.227359	0.100018	0.100050	0.001200	0.001206	0.012045
1.017734	2.164576	0.206275	0.100014	0.100013	0.001200	0.001201	0.013900
1.001558	2.675293	0.225136	0.100004	0.100036	0.001200	0.001210	0.012360
1.006552	4.035427	0.239247	0.100003	0.100196	0.001200	0.001215	0.012382
1.004194	2.959261	0.227359	0.100018	0.100050	0.001200	0.001206	0.012045
1.000507	2.690279	0.226175	0.100004	0.100036	0.001200	0.001216	0.012360
1.019715	3.056484	0.225642	0.100007	0.100005	0.001200	0.001214	0.013397
1.000052	4.049549	0.234708	0.100016	0.100052	0.001200	0.001208	0.012587
1.004060	2.905545	0.227359	0.100016	0.100042	0.001200	0.001200	0.012048
1.001932	3.785636	0.234708	0.100014	0.100020	0.001200	0.001210	0.012630
1.024291	2.666478	0.226455	0.100011	0.100036	0.001200	0.001201	0.012204
1.004060	2.905345	0.227359	0.100016	0.100029	0.001200	0.001200	0.011855
1.000525	2.797113	0.226245	0.100006	0.100008	0.001200	0.001215	0.011952
1.001348	3.103129	0.226827	0.100026	0.100016	0.001200	0.001207	0.012636
1.006974	3.691453	0.237983	0.100005	0.100111	0.001200	0.001210	0.012382
1.001932	3.785636	0.234708	0.100014	0.100018	0.001200	0.001210	0.012630
1.011982	4.070014	0.240088	0.100013	0.100226	0.001200	0.001207	0.012641
1.001891	3.634840	0.210869	0.100007	0.100955	0.001200	0.001215	0.019953

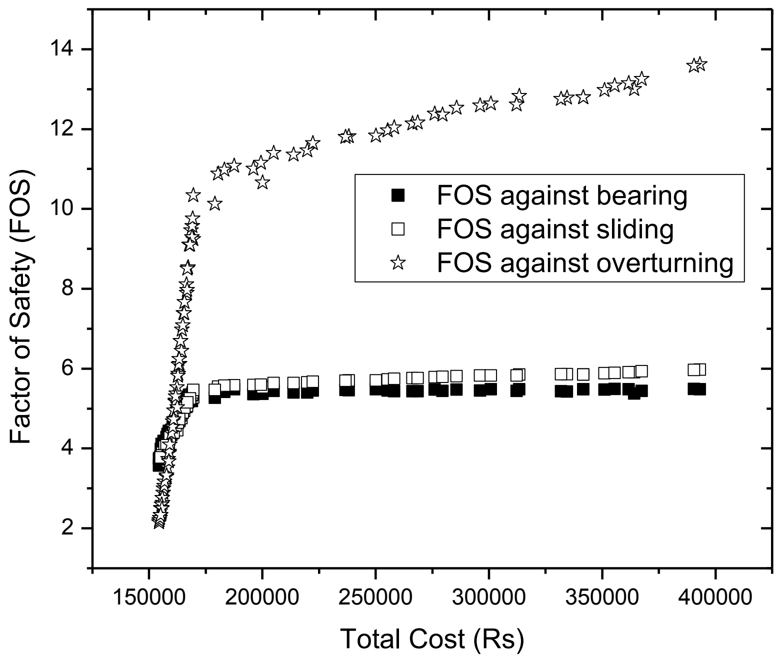

Fig. 19. Variation of factor of safety with total cost against bearing, sliding and overturning Figure for the RCC RW of height 9m and ϕ of 40^0

It is evident that due to in increase in ϕ value, the total cost decreased. There is substaintial increase in the FOS against overturning with increase in Total Cost, however there is similar trend for effect of FOS against bearing and sliding with increase in Total Cost.

Table 13 The dimensions of the retaining wall and the percentage of reinforcement for the RW of 10m and ϕ of 20^0

L_t	L_h	t	S	b	P_{ts}	P_{th}	P_{tt}
2.051	2.995	0.912	0.926	0.221	0.826	0.406	0.363
2.051	2.998	0.912	0.926	0.224	0.826	0.406	0.363
2.071	2.999	0.921	0.926	0.181	0.826	0.406	0.363
2.094	2.998	0.913	0.927	0.222	0.826	0.406	0.363
2.343	2.98	0.902	0.93	0.188	0.82	0.358	0.485
2.359	2.98	0.906	0.93	0.188	0.82	0.358	0.485
2.38	2.98	0.9	0.933	0.15	0.83	0.37	0.485
2.452	2.996	0.906	0.926	0.188	0.834	0.36	0.485
2.48	2.997	0.914	0.923	0.184	0.834	0.36	0.485
2.871	3	0.947	0.931	0.113	0.823	0.258	0.468
2.894	2.997	0.952	0.931	0.084	0.818	0.275	0.471
2.909	2.999	0.946	0.931	0.108	0.818	0.275	0.473
2.949	3	0.939	0.932	0.13	0.82	0.249	0.486
3.005	3	0.95	0.932	0.085	0.82	0.252	0.481
3.014	3	0.944	0.932	0.115	0.82	0.252	0.481
3.178	2.997	1	0.927	0.965	2.603	0.424	2.346
3.182	2.997	0.986	0.934	0.95	0.941	0.176	0.503
3.196	3	0.966	0.942	0.915	0.796	0.125	0.455
3.196	3	0.966	0.942	0.915	0.796	0.124	0.536
3.205	2.998	0.973	0.926	0.745	0.826	0.135	0.334
3.205	2.998	0.991	0.926	0.745	0.826	0.135	0.334
3.212	3	0.994	0.931	0.076	0.822	0.214	0.422
3.232	3	0.998	0.947	0.791	0.821	0.152	0.287
3.243	3	0.95	0.977	0.763	0.757	0.121	0.523
3.246	3	0.998	0.947	0.791	0.821	0.152	0.287
3.255	3	0.971	0.789	0.996	1.497	0.348	1.357
3.26	2.999	0.944	0.78	0.967	1.294	0.203	0.682
3.263	2.996	0.999	0.917	0.873	2.428	0.397	1.686
3.265	2.996	0.993	0.918	0.873	0.883	0.397	1.641
3.266	2.998	0.932	0.778	0.983	2.825	0.139	0.53
3.266	3	0.947	0.772	0.984	2.673	0.139	0.369
3.266	2.998	0.944	0.778	0.987	2.673	0.139	0.561

3.266	2.998	0.932	0.778	0.983	2.825	0.139	0.53
3.271	3	0.998	0.948	0.791	0.821	0.154	0.362
3.275	3	0.941	0.779	0.967	1.362	0.24	1.248
3.276	2.999	1	0.81	0.96	2.173	0.146	1.509
3.278	2.999	0.99	0.807	0.962	2.29	0.323	1.617
3.279	3	0.95	0.987	0.77	1.112	0.121	0.523
3.28	2.999	1	0.807	0.962	2.261	0.375	1.671
3.285	2.998	0.995	0.778	0.983	2.866	0.3	0.562
3.289	2.997	0.992	0.807	0.952	2.752	0.169	1.849
3.29	3	0.952	0.957	0.789	2.774	0.512	0.521
3.292	3	0.996	0.811	0.927	1.375	0.323	1.224
3.293	2.997	0.984	0.934	0.818	0.957	0.176	0.503
3.435	2.989	0.944	0.997	0.415	0.698	0.189	0.454
3.54	3	0.929	0.932	0.353	0.82	0.175	0.486
3.544	2.995	0.945	0.949	0.05	0.783	0.202	0.505
3.551	3	0.945	0.95	0.051	0.783	0.202	0.505
3.556	2.989	0.944	0.997	0.435	0.713	0.179	0.454
3.571	2.989	0.95	0.997	0.435	0.713	0.179	0.454
3.58	3	0.945	0.95	0.051	0.783	0.202	0.505
3.636	3	0.929	0.932	0.351	0.82	0.153	0.486
3.638	2.999	0.945	0.95	0.051	0.783	0.202	0.505
3.649	3	0.945	0.95	0.051	0.783	0.202	0.505
3.652	3	0.929	0.932	0.355	0.82	0.175	0.486
3.655	3	0.952	0.95	0.056	0.783	0.18	0.505
3.661	3	0.94	0.932	0.364	0.82	0.154	0.521
3.692	3	0.951	0.95	0.055	0.783	0.177	0.505
3.749	3	0.948	0.95	0.051	0.782	0.183	0.505
3.75	3	0.943	0.95	0.051	0.782	0.184	0.505
3.757	3	0.941	0.947	0.074	0.782	0.191	0.503
3.815	3	0.945	0.95	0.051	0.782	0.183	0.505
3.816	3	0.947	0.95	0.087	0.782	0.168	0.505
3.822	3	0.949	0.947	0.072	0.782	0.19	0.498
3.866	3	0.949	0.947	0.074	0.782	0.19	0.498

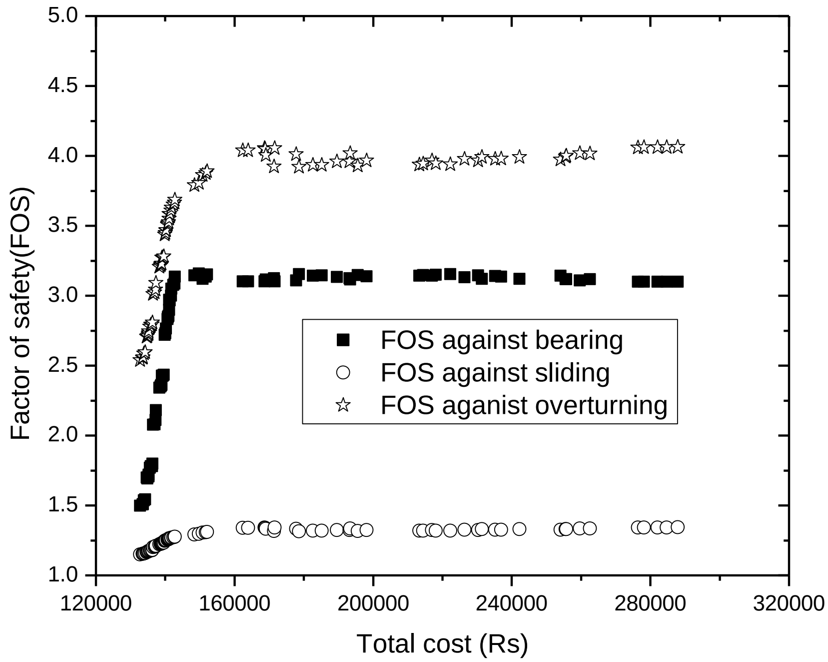

Fig. 20. Variation of factor of safety with total cost against bearing, sliding and overturning Figure For the RCC RW of height 10m and ϕ of 20^0

It can be seen that the optimum cost is different, but the trend in variations of total cost with the FOS are similar. It can also be seen that there is an optimum point upto which the FOS can be increased with the cost, but then after the increase in dimension of the retaining wall increased the cost, but with minor changes in the FOS value.

It was found in the table that there are some points very close to each other as part of the Pareto set. This is due to the fact that in case of a population based approach, few solutions may be very close. It can be seen that in comparison to L_h value there is wide variation in L_t value (2.05 to 3.86). Similarly, the thickness of base slab is almost constant, but there is a change in the 'b' corresponding to increase in cost and FOS value. To compare the effect of angle of internal friction value, the variation of cost and FOS for a retaining wall of 10m height and $\phi = 40^0$ is shown in Fig. 22.

Table 14 The dimensions of the retaining wall and the percentage of reinforcement for the RW of 10m and ϕ of 40^0

L_t	L_h	t	S	b	P_{ts}	P_{th}	P_{tt}
1.032	2.840	0.437	0.721	0.054	0.577	0.672	0.519
1.013	2.690	0.437	0.721	0.054	0.577	0.672	0.526
1.416	3.000	0.558	0.962	0.575	0.314	0.158	0.266
1.745	2.988	0.486	0.757	0.081	0.522	0.313	0.753
1.786	2.993	0.486	0.756	0.100	0.522	0.302	0.753
1.786	2.994	0.486	0.756	0.142	0.522	0.302	0.753
1.147	2.999	0.432	0.994	0.999	0.308	0.243	0.377
1.619	3.000	0.559	0.937	0.229	0.337	0.266	0.387
1.046	2.962	0.437	0.721	0.055	0.577	0.672	0.529
1.573	3.000	0.559	0.952	0.164	0.323	0.209	0.371
1.313	3.000	0.432	0.918	0.763	0.345	0.264	0.467
1.003	2.804	0.451	0.691	0.051	0.643	0.639	0.508
1.237	2.884	0.444	0.721	0.062	0.577	0.551	0.731
1.282	2.954	0.440	0.722	0.050	0.577	0.586	0.793
1.014	2.828	0.438	0.721	0.053	0.577	0.664	0.508
1.416	3.000	0.558	0.962	0.528	0.314	0.158	0.266
1.090	3.000	0.423	0.721	0.056	0.577	0.715	0.689
1.065	2.922	0.438	0.721	0.052	0.577	0.672	0.548
1.350	3.000	0.452	0.938	0.705	0.345	0.237	0.566
1.026	2.818	0.437	0.721	0.054	0.577	0.672	0.519
1.423	2.958	0.443	0.718	0.052	0.593	0.496	0.892
1.416	3.000	0.558	0.962	0.546	0.314	0.168	0.266
1.533	3.000	0.556	0.953	0.360	0.317	0.186	0.329
1.248	2.998	0.434	0.971	0.804	0.304	0.301	0.488
1.123	2.999	0.365	0.984	0.994	0.305	0.320	0.523
1.676	2.998	0.506	0.764	0.210	0.521	0.358	0.658
1.003	2.699	0.451	0.691	0.053	0.643	0.639	0.508
1.530	3.000	0.536	0.967	0.379	0.317	0.211	0.388
1.090	3.000	0.423	0.721	0.051	0.577	0.715	0.675
1.123	3.000	0.365	0.971	0.979	0.304	0.301	0.517
1.003	2.699	0.451	0.691	0.051	0.643	0.639	0.508
1.423	2.990	0.443	0.718	0.050	0.605	0.502	0.892
1.752	2.987	0.491	0.757	0.092	0.522	0.313	0.753
1.288	2.877	0.428	0.722	0.050	0.577	0.582	0.877
1.013	2.742	0.437	0.721	0.054	0.577	0.672	0.526
1.061	2.922	0.436	0.721	0.052	0.577	0.672	0.544
1.684	2.999	0.558	0.928	0.060	0.340	0.226	0.451

1.660	3.000	0.518	0.929	0.244	0.357	0.236	0.631
1.681	2.997	0.496	0.755	0.053	0.523	0.342	0.710
1.090	3.000	0.422	0.721	0.051	0.577	0.715	0.652
1.675	3.000	0.599	0.939	0.108	0.326	0.203	0.383
1.013	2.745	0.437	0.721	0.057	0.577	0.672	0.526
1.684	2.999	0.558	0.928	0.051	0.340	0.226	0.451
1.032	2.879	0.437	0.721	0.054	0.577	0.672	0.519
1.423	2.949	0.443	0.718	0.052	0.593	0.496	0.892
1.123	3.000	0.365	0.971	0.983	0.304	0.301	0.517
1.029	2.848	0.438	0.721	0.053	0.577	0.664	0.512
1.634	2.963	0.522	0.952	0.297	0.324	0.266	0.443
1.331	2.998	0.437	0.726	0.053	0.577	0.626	0.820
1.675	3.000	0.582	0.957	0.119	0.326	0.203	0.383
1.730	3.000	0.528	0.755	0.210	0.521	0.325	0.658
1.003	2.575	0.448	0.691	0.051	0.643	0.639	0.508
1.224	2.999	0.988	0.989	0.999	0.503	0.448	0.275
1.003	2.690	0.437	0.721	0.054	0.577	0.672	0.526
1.003	2.575	0.448	0.691	0.050	0.643	0.639	0.508
1.203	3.000	0.814	1.000	1.000	0.287	0.124	0.298
1.220	2.999	0.450	0.721	0.051	0.577	0.579	0.633
1.248	2.998	0.434	0.984	0.804	0.305	0.319	0.440
1.158	2.998	0.452	0.721	0.050	0.577	0.620	0.560
1.209	3.000	0.897	0.999	0.999	0.293	0.122	0.567
1.717	3.000	0.489	0.713	0.056	0.593	0.395	0.829
1.324	2.986	0.439	0.722	0.050	0.577	0.579	0.869
1.158	2.998	0.452	0.721	0.050	0.577	0.615	0.560
1.657	2.987	0.472	0.697	0.053	0.624	0.395	0.825
1.533	3.000	0.584	0.996	0.370	0.292	0.177	0.294
1.003	2.656	0.448	0.691	0.050	0.643	0.639	0.508
1.657	2.999	0.558	0.928	0.052	0.340	0.226	0.433
1.573	3.000	0.559	0.981	0.164	0.323	0.209	0.371
1.530	3.000	0.536	0.964	0.379	0.317	0.211	0.388
1.308	2.999	0.440	0.722	0.050	0.577	0.550	0.793
1.004	2.656	0.448	0.691	0.050	0.643	0.639	0.508
1.752	2.987	0.491	0.757	0.056	0.522	0.312	0.753
1.786	2.993	0.486	0.756	0.103	0.522	0.302	0.753
1.684	2.999	0.558	0.930	0.075	0.340	0.226	0.451
1.129	2.998	0.706	1.000	1.000	0.287	0.120	0.131
1.719	2.999	0.454	0.713	0.053	0.593	0.395	0.979
1.282	2.954	0.437	0.722	0.050	0.577	0.586	0.793
1.740	2.999	0.560	0.933	0.075	0.340	0.210	0.455

1.224	2.999	0.757	0.989	0.961	0.581	0.577	0.275
1.153	2.999	0.432	0.987	0.999	0.308	0.243	0.377
1.158	2.999	0.689	0.975	0.998	0.317	0.131	0.130
1.030	2.922	0.439	0.721	0.052	0.577	0.672	0.512
1.231	3.000	0.991	0.993	1.000	0.410	0.600	0.482
1.191	2.997	0.792	1.000	0.986	0.287	0.120	0.142
1.203	3.000	0.814	1.000	1.000	0.287	0.124	0.298
1.220	2.999	0.450	0.721	0.050	0.577	0.579	0.633
1.775	2.999	0.560	0.933	0.075	0.340	0.210	0.455
1.151	2.996	0.743	1.000	1.000	0.287	0.120	0.142
1.686	2.999	0.454	0.713	0.055	0.593	0.395	0.979
1.678	2.998	0.473	0.697	0.054	0.624	0.395	0.820
1.209	2.994	0.855	1.000	0.999	0.293	0.643	0.523
1.717	3.000	0.489	0.713	0.056	0.593	0.395	0.829
1.225	3.000	0.970	0.999	1.000	0.633	0.534	0.727
1.209	3.000	0.980	0.999	1.000	0.293	0.122	0.568
1.660	3.000	0.518	0.956	0.244	0.357	0.236	0.603
1.225	3.000	0.970	0.999	1.000	0.522	0.594	0.475
1.248	2.999	0.458	0.986	0.819	0.360	0.351	0.370
1.225	3.000	0.993	1.000	1.000	2.263	1.031	0.420
1.248	2.998	0.434	0.984	0.731	0.305	0.319	0.440
1.032	2.879	0.437	0.721	0.054	0.577	0.672	0.519

In comparison to Table 13, it can be seen that the dimensions like Lt corresponding to $\phi = 40^0$ is less than half of that corresponding to $\phi = 20^0$, but the variation in Lh is very marginal. Similarly, though the thickness of the base slab reduced, the variation in thickness of stem slab is very less. Such a study gives the opportunity to the professional engineers not only to choose the suitable FOS and the corresponding cost, but also the corresponding footing dimensions and percentage of reinforcement.

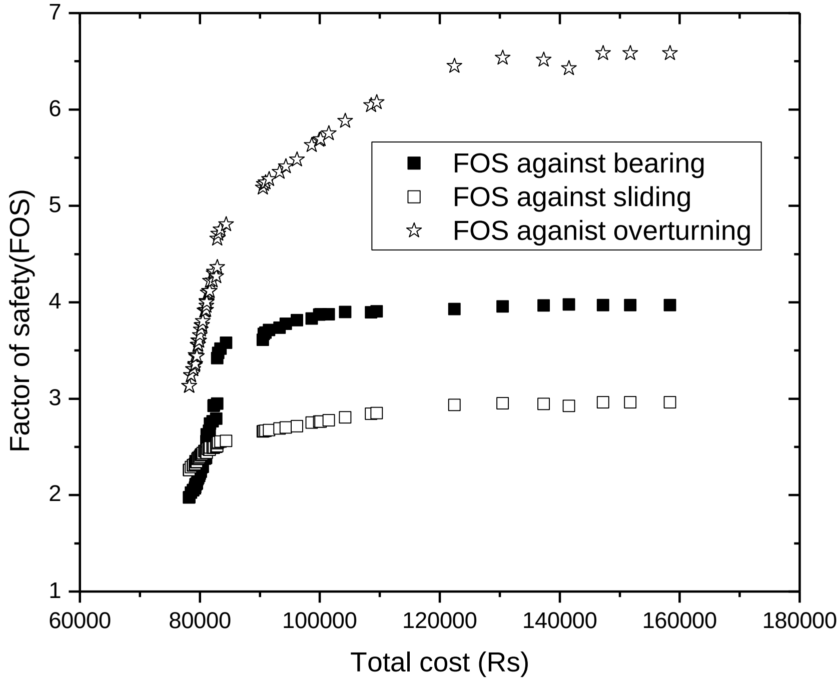

Fig. 21. Variations of factor of safety with total cost against bearing, sliding and overturning for the RCC RW of height 10m and ϕ of 40^{02}

It can be seen that due to increase in ϕ value, the total cost reduced, but the trend is similar with FOS against sliding is critical. It can be mentioned here that, the increase in FOS against sliding is very less compared to FOS bearing and overturning.

Chapter 4| CONCLUSIONS

The present study discussed about the optimum design of RCC cantilever retaining wall in a single and multi-objective framework using genetic algorithm, NSGA-II. Based on the results and discussion thereof, following conclusions can be made.

A set of effective Pareto optimal set was observed, indicating the efficacy of NSGA-II in finding out distinct and number of Pareto solutions. It was observed that there is a steady increase in FOS against bearing to increase in cost upto FOS 4.0 , then after the FOS does not change appreciably with increase in cost. It was also found that FOS against sliding is the controlling factor for the considered retaining wall.

It was also observed that in the Pareto solutions, there is wide variation in Lt value (2.05 to 3.86), though variation in L_h value is marginal. Similarly, the thickness of base slab is almost constant, but there is a change in the 'b' corresponding to increase in cost and FOS value.

It was observed that for a wall height of 10m, the dimension like Lt corresponding to $\phi = 40^0$ is less than half of that corresponding to $\phi = 20^0$, but the variation in Lh is very marginal. Similarly, though the thickness of the base slab reduced, the variation in thickness of stem slab is very less.

Such a study gives ample opportunities to the professional engineers not only to choose the suitable FOS and the corresponding cost, but also the corresponding footing dimensions and percentage of reinforcement.

[1] Ahmadi-Nedushan, B., and Varaee, H. Optimal design of reinforced concrete retaining walls using a swarm intelligence technique. Proc., 1st Int. Conf. on Soft computing Technology in Civil, Structural and Environmental Engineering, Civil-Comp Press, Stirlingshire, U.K; 2009.

[2] Alshawi, F. A. N., Mohammed, A. I., and Farid, B. J. Optimum design of tied-back retaining walls. Struct. Eng. (London), 1988; 66(6):97–105.

[3] Babu, G.L.S., and Basha, B.M. Optimum Design of Cantilever Retaining Walls Using target Reliability Approach. International Journal of Geomechanics, (ASCE) 2008; 8(4):240–52.

[4] Bhatti, M. A. Retaining wall design optimization with MS excel solver. Proc., 17th Analysis and Computation Specialty Conf., (ASCE) 2006, Reston, VA.

[5] Camp, C.V., and Akin, A. Design of Retaining Walls Using Big Bang–Big Crunch Optimization. J. Struct. Eng., 2012; 138: 438-48

[6] Camp, C. V., Pezeshk, S., and Hansson, H. Flexural design of reinforced concrete frames using a genetic algorithm. J. Struct.Eng., 2003; 129(1):105–15.

[7] Castillo, E., Mínguez, R., Ruiz-Terán, A., and Fernández-anteli, A. Design and sensitivity analysis using the probability-safety factor method: An application to retaining walls. Struct. Safety, 2004; 26(2):159–79.

[8] Ceranic, B., Fryer, C., and Baines, R. W. An application of simulated annealing to the optimum design of reinforced concrete retaining structures. Comput. Struct., 2001; 79(17):1569–81.

[9] Chau, K. W., and Albermani, F. Knowledge-based system on optimum design of liquid retaining structures with genetic algorithms. J. Struct. Eng., 2003; 129(10):1312–21.

[10] Coello C A C, Veldhuizen D A V, Lamont G B. Evolutionary algorithms for solving multi-objective problems. New York: Kluwer Academic Publishers; 2002.

[11] Coello Coello, C., Christiansen, A. D., and Santos, F. A simple genetic algorithm for the design of reinforced concrete beams. Eng.Comput., 1997; 13(4):185–96.

[12] Deb, K. Multi-objective optimization using evolutionary algorithms. Wiley, Chichester, UK; 2001.

[13] Deb K, and Dhar A. Optimum Design of Stone Column-Improved Soft Soil Using Multiobjective Optimization Technique. Computers and Geotechnics, 2011; 38(1):50 – 7.

[14] Deb K, Dhar A, and Bhagat P. Evolutionary Approach for Optimal Stability Analysis of Geosynthetic-Reinforced Stone Column-Supported Embankments on Clay. KSCE Journal of Civil Engineering, 2012; 16(7):1185-92.

[15] Delhi Schedule of Rates (2007)., CPWD, Government of India.

[16] Fang, Y. S., Chen, T. J., and Wu, B. F. Passive earth pressures with various wall movements. J Geotech. Eng. (ASCE) 1994; 120(8):1307-23.

[17] Govindaraj, V., and Ramasamy, J. V. Optimum detailed design of reinforced concrete continuous beams using genetic algorithms. Comput. Struct., 2005; 84(1–2):34–48.

[18] Kaveh, A., and Abadi, A. S. M. Harmony search based algorithm for the optimum cost design of reinforced concrete cantilever retaining walls. Int. J. Civ. Eng., 2010;9(1):1–8.
[19] Keskar, A. V., and Adidam, S. R. Minimum cost design of a cantilever retaining wall. Indian Concrete Journal, 1989; 63(8):401-05.
[20] Kwak, H. G., and Kim, J. Optimum design of reinforced concrete plane frames based on predetermined section database. Comput. Aided Des., 2008; 40(3):396–408.
[21] Kwak, H. G., and Kim, J. An integrated genetic algorithm complemented with direct search for optimum design of RC frames. Comput. Aided Des., 2009; 41(7):490–500.
[22] Lee, C. L., and Ahn, J. Flexural design of reinforced concrete frames by genetic algorithm. J. Struct. Eng., 2003; 129(6):762–74.
[23] Lepš, M., and Šejnoha, M. New approach to optimization of reinforcedconcrete beams. Comput. Struct., 2003; 81(18–19):1957–66.
[24] Low, B. K. Reliability-based design applied to retaining walls. Geotechnique, 2005;55(1):63-75.
[25] Low, B. K., Teh, C. I., and Tang, W. H. Efficient reliability-based design using spreadsheet optimization. Proc. 8th Int. Conf. on Structural Safety and Reliability, ICOSSAR, A. A. Balkema Publishers, Rotterdam, The Netherlands; 2001.
[26] Paya, I., Yepes, V., González-Vidosa, F., and Hospitaler, A. Multi-objective optimization of concrete building frames by simulated annealing. Comput. Aided Civ. Infrastruct. Eng., 2008; 23(8):596–610.
[27] Paya-Zaforteza, I., Yepes, V., Hospitaler, A., and González-Vidosa, F. CO2-optimization of reinforced concrete frames by simulated annealing. Eng. Struct., 2009; 31(7):1501–08.
[28] Perea, C., Alcala, J., Yepes, V., González-Vidosa, F., and Hospitaler, A. Design of reinforced concrete bridge frames by heuristic optimization. Adv. Eng. Software, 2008; 39(8):676–88.
[29] Rafiqa, M. Y., and Southcombea, C. Genetic algorithms in optimal design and detailing of reinforced concrete biaxial columns supported by a declarative approach for capacity checking. Comput.Struct., 1998; 69(4):443–57.
[30] Rajeev, S., and Krishnamoorthy, C. S. Genetic algorithms-based methodology for design optimization of reinforced concrete frames. Comput. Aided Civ. Infrastruct. Eng., 1998; 13(1):63–74.
[31] Rhomberg, E. J., and Street, W. M. Optimal design of retaining walls. J. Struct. Div. (ASCE) 1981;107(5):992–1002.
[32] Sahab, M. G., Ashour, A. F., and Toropov, V. V. Cost optimisation of reinforced concrete flat slab buildings. Eng. Struct., 2004; 27(3):313–22.
[33] Saribas, A., and Erbatur, F. Optimization and sensitivity of retaining structures. J. Geotech. Engrg., 1996; 122(8):649–56.
[34] Srinivas, N., and Deb, K. Multi-Objective function optimization using non-dominated sorting genetic algorithms. Evolutionary Computation Journal, 1994; 2(3):221-48.
[35] Wang, Y., and Kulhawy, F. H. Economic design optimization of foundations. Journal of Geotechnical and Geoenvironmental Engineering, 2008; 134(8):1097-1105.
[36] Yepes, V., Alcala, J., Perea, C., and González-Vidosa, F. A parametric study of optimum earth-retaining walls by simulated annealing. Eng. Struct., 2008; 30(3):821–30.